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ANALYSES OF UNATTENDED GROUND SENSORS IN THEATER MISSILE DEFENSE ATTACK OPERATIONS

by

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ANALYSES OF UNATTENDED GROUND SENSORS IN THEATER MISSILE DEFENSE ATTACK OPERATIONS

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ABSTRACT

Unattended ground sensors have a tremendous potential for improving Tactical Ballistic Missile Attack Operations. To date, however, this potential has gone unrealized primarily due to a lack of confidence in the systems and a lack of tactical doctrine for their employment. This thesis provides analyses to demonstrate the effective use of sensor technology and provides recommendations as to how it may best be employed.

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LIST OF ABBREVIATIONS

AOR	Area of Responsibility
CAP	Combat Air Patrol
CPA	Closest Point of Approach
kph	Kilometers Per Hour
OOB	Order of Battle
RPV	Remotely Piloted Vehicle
SAM	Surface to Air Missile
SOF	Special Operations Force
UAV	Unmanned Aerial Vehicle
UGS	Unattended Gound Sensor
TBM	Theater Ballistic Missile
TBMD	Theater Ballistic Missile Defense
TCT	Time Critical Target
TEL	Transporter-erector Launcher

LIST OF SYMBOLS

P_d	Single incident Bernoulli trial probability sensor detects a target within its maximum range.
f_1	Probability of identification by sensor.
f_0	Probability of mis-identification by sensor.
t_s	Sensor sample rate (samples/ second).
v	Vehicle velocity (kilometers per hour).
R_{max}	Maximum sensor range (meters).
r	Maximum effective sensor range corrected for off-road distance (meters).
d_s	Perpendicular off-road distance (meters).
d_t	Distance traveled between sensor samples (meters).
s	Number of samples available per sensor.
$P\{\text{detect} \mid t_s\}$	Probability sensor detects passing vehicle for given sample rate.
P_{detect}	Probability sensor detects passing vehicle given sensor technology (P_d, R_{max}, t_s) and operational considerations (v, d_s).
P_x	Fraction of vehicles that are TCTs.
$P\{\text{TCT}\mid k/n\}$	Probability detected vehicle is a TCT given exactly k of n sensors in the array indicate it is.
$P\{\text{TCT}\mid \geq k/n\}$	Probability detected vehicle is a TCT given at least k of n sensors in the array indicate it is.
$P\{\text{TCT}\}$	Probability detected vehicle is TCT based on forecast by array.
N	Number of sensors available to theater commander.
A	Number of candidate array sites.
i	Index of a road to be seeded with an array.
$P_x(i)$	Fraction of traffic on road i comprised of TCTs.
λ_i	Vehicle flow rate on road i (vehicles per hour).
n_i^*	Optimal array size for road i.
t_R	Theater commander's maximum desired reporting time for entire array.

P_c	Probability one or more sensors are compromised, given one sensor is found.
k_i^*	Minimum number of sensors indicating vehicle is a TCT for the theater commander to commit an attack asset.
t	Search time for hostile force (minutes).
m	Number of searchers in hostile force.
S	Searcher speed (m^2/ second).
A_s	Area searched by m individuals each searching at speed S for t minutes (m^2).
V_{TCT}	Transiting time critical target velocity (kph).
w_1	Theater Commander's weight on reporting time.
w_2	Theater Commander's weight on compromise.
w_3	Theater Commander's weight on countermeasures.

EXECUTIVE SUMMARY

Unattended ground sensors have a tremendous potential for improving Tactical Ballistic Missile Attack Operations. To date, however, this potential has gone unrealized primarily due to a lack of confidence in the systems and a lack of tactical doctrine for their employment. This thesis provides analyses to persuade decision makers to trust sensor technology and provides recommendations as to how they may best be employed.

A Time Critical Target (TCT) is any military vehicle which, from its standard tactics, can be expected to remain on the road system for a period not to exceed 30 minutes. As a result of this, actions taken to prosecute this type of target must be made expeditiously. TCT's include more than just Transporter-erector Launcher (TEL) units. They may also be command vehicles, missile fuel trucks, missile loading trailers, and mobile SAM units, among others.

Theater ballistic missile (TBM) defense basically follows two discrete doctrines. Counterforce is the destruction of a TCT as it travels to and from its assembly area for reloading and maintenance. Active defense is the destruction of the individual tactical ballistic missile after launch during the boost, reentry or terminal phases. Only the counterforce defense strikes the target while it is slow-moving and, more importantly, only counter force prevents future use of the same TCT.

In the pre-hostility phase of a conflict, it will become necessary to locate and monitor TBM vehicles to prepare an adequate counter-attack should the conflict escalate. In addition to national collection assets, unattended ground sensors, placed strategically along known or suspected TCT travel routes, would aid in the development of a clear tactical picture to meet this end.

Should hostilities escalate to the point where armed response is required, a prime concern of the theater commander in the early phase of the conflict is the enemy TBM threat. It is in this early phase that the enemy has the greatest chance of surprise, and the full strength of its TBM force. TCT routes confirmed in the pre-hostility phase by ground sensors can now be covered by combat air patrol (CAP) or lethal unmanned aerial vehicles

(UAV) awaiting a targeting order. Should a sensor array indicate the presence of a TCT, the decision maker can order the prosecution of that target by assets already on station.

The analyses provided in this document emphasize the use of unattended ground sensors in the locating and positive identification of time critical targets. Some degree of friendly intelligence capability is assumed in that candidate sensor array sites are chosen within the theater, and realistic enemy TCT populations are used. With the arrays in place, the decision maker must be able to evaluate the output of the array to determine, with some degree of confidence, if the target before the array is a TCT or a false indication.

The probabilistic decision model is based on there being a finite number of sensors available in a specific theater, with a fixed number of pre-determined candidate array sites. Further inputs include the approximate fraction of vehicle traffic assumed to be TCT on each road to be seeded. This assumption is based on the belief that the intelligence analysts have pre-determined the most likely areas for TBM activity.

From the above input data, the model reports the optimal size array for each of the candidate locations. Additionally, the decision maker is provided with an optimal policy for determining the likelihood that the target is a TCT based on the number of sensors in agreement as to its identity. This policy may vary with each candidate array.

Finally, recommendations are provided on the placement of the arrays and their best configuration to maximize information gained while minimizing the likelihood of compromise. Specifics include, inter-sensor spacing, placement patterns, array locations, and off-road distance.

Theater ballistic missiles are a significant threat to any theater commander in the battlefields of the future. It is only through the optimal use of all available sensors that decisive action may be taken against these time critical targets. Unattended ground sensors, when properly deployed, provide an inexpensive reliable option to monitor the battlespace before, during and after the conflict.

I. INTRODUCTION

Unattended ground sensors have a tremendous potential for improving Tactical Ballistic Missile Attack Operations. To date, however, this potential has gone unrealized primarily due to a lack of confidence in the systems and a lack of tactical doctrine for their employment. This thesis provides analyses to demonstrate the effective use of sensor technology and provides recommendations as to how it may best be employed. The relatively disappointing show of active defense systems, such as Patriot, has illustrated the need for a more reliable, yet cost-effective means of defending theater forces. Further, it has been shown that forces must be successful in attack operations, without which active defense will never be enough.

Since the 1960's, ground sensors have been used to sharpen the theater tactical picture with limited effect. An early attempt, the McNamara Line of 1967, was aimed at slowing the flow of military goods along the Ho Chi Minh Trail by identifying targets for air strikes. These were arcane acoustic and seismic detectors with relaying aircraft, both manned and remotely piloted, used to monitor the output. The normal time between target acquisition and weapon delivery was approximately five minutes, yet few kills were confirmed[Ref. 1]. These poor results have been attributed to the unreliable and limited output of the sensors. Specifically, the sensors of the McNamara Line could determine if a target was personnel, wheeled vehicle, or tracked vehicle. For the United States, the losses fighting the sensor war were significant. Of the more than 600 planes and helicopters lost in Laos, one half to two-thirds were lost to defensive positions along the Ho Chi Minh Trail [Ref. 1]. Today's mission is similar, but the sensors have improved tenfold. The key, however, remains detecting and identifying a viable military target against which minimal assets are directed.

A Time Critical Target (TCT) is any military vehicle which, from its standard tactics, can be expected to remain on a road system for a period not to exceed 30 minutes. As a result, actions taken to prosecute this type of target must be made expeditiously. TCTs include Transporter-erector Launcher (TEL) units, command vehicles, missile fuel trucks, missile loading trailers, and mobile SAM units, among others. During the Gulf

War American pilots used random search tactics in an attempt to locate Iraqi TELs. Not only did this tactic prove largely unsuccessful, but it required the coalition to completely control the airspace above the battlefield. A more prudent tactic would be to send friendly aircraft over hostile territory only to prosecute a specific target already detected, located and identified by electronic means.

A. GENERAL BACKGROUND

Theater ballistic missile (TBM) defense basically follows two discrete doctrines. Counterforce is aimed at the destruction of a TCT as it travels to and from its assembly area for reloading and maintenance. Active defense is aimed at the destruction of the individual tactical ballistic missile after launch during the boost, reentry or terminal phases. Only the counterforce strikes the target while it is slow-moving and, more importantly, only counterforce prevents future use of the same TCT and its crew. It has been shown that counterforce, even in modest proportions, geometrically reduces the incoming tactical ballistic missile threat [Ref. 2].

In the pre-hostility phase of a conflict, it will become necessary to locate and monitor TBM vehicles to prepare an adequate counter-attack should the conflict escalate. In addition to national collection assets, unattended ground sensors, placed strategically along known or suspected TCT travel routes, would aid in the development of a clear tactical picture to meet this end. Special Operations forces are also ideally suited for this type of work, but they constitute a limited and valuable commodity. Laying the sensor arrays, however, would definitely be a part of the pre-conflict Special Operations repertoire.

Should hostilities escalate to the point where armed response is required, a prime concern of the theater commander in the early phase of the conflict is the enemy TBM threat. It is in this early phase that the enemy has the greatest chance of surprise, and the full strength of its TBM force. TCT routes confirmed in the pre-hostility phase by ground sensors can now be covered by combat air patrol (CAP) or lethal unmanned aerial vehicles (UAV) awaiting a targeting order. Should a sensor array indicate the presence of a TCT, the theater commander can order the prosecution of that target by assets already on station. Turnaround times would be similar to those of the McNamara Line, but only against mission essential targets. Additionally, unlike the Gulf War, aircraft would be used for strike only, not for search. This would free additional sorties for the CAP missions.

There is also a political advantage to preparing a battlefield with unattended ground sensors. Through CNN America saw technology and training win a nearly bloodless battle in the Gulf War. Continuing this trend will require the reduction in the number of humans on the battlefield. Simply put,

> There would be far less anger directed against an encounter in which the United States was putting hardware, not men, on the line and one which American casualty lists were dominated by decimated sensors, burnt-out computers, and downed RPVs [Ref. 3].

America's growing need for instant access to newsworthy information and demand for quickly resolved conflicts will require commanders to keep theater conflicts short and neat. It may be neither feasible nor desirable to await the final breakdown of negotiations before pinpointing enemy TBM forces. The use of unattended ground sensors could aid in defining hostile depots, staging areas, and hide sites prior to hostilities.

B. ANALYSES

The analyses provided in this document emphasize the use of unattended ground sensors in the locating and positive identification of time critical targets. Some degree of friendly intelligence capability is assumed in that candidate sensor array sites are chosen within the theater, and realistic enemy TCT populations are used. With the arrays in place, the decision maker must be able to evaluate the output of the array to determine, with some degree of confidence, if the target before the array is a TCT or a false indication.

The probabilistic decision model is developed and described in Chapter II. It is based on there being a finite number of sensors available in a specific theater, with a fixed number of pre-determined candidate array sites. Further inputs include the approximate fraction of vehicle traffic assumed to be TCTs on each road to be seeded. This assumption is based on the belief that the intelligence analysts have pre-determined the most likely areas for TBM activity.

From the above input data, the model reports the optimal array size for each of the candidate locations. Additionally, the decision maker is provided with an optimal policy for determining the likelihood that the target is a TCT based on the number of sensors in agreement as to its identity. This policy may vary with each candidate array.

In Chapter III, recommendations are provided on the placement of the arrays and their best configuration to maximize information gained while minimizing the likelihood of compromise. Specifics addressed include, inter-sensor spacing, placement patterns, array locations, and off-road distance.

Chapter IV contains an analysis of the optimal location for the sensor array, along a given road. Considerations include locating at road intersections, along roads away from intersections, and at choke points through which roads pass.

Finally, Chapter V contains conclusions on all the analyses performed and provides a step-by-step procedure for defining, locating, and utilizing sensor arrays in a theater of operations.

II. MODELS AND ASSUMPTIONS

A. INTRODUCTION

In the quest to determine the optimal sensor deployment scheme, it is most important to choose a model representative of actual Theater Ballistic Missile Defense (TBMD) operations. After some important clarifications, this chapter introduces the two separate models used in this thesis.

First it is necessary to introduce the reader to the terminology common to TBM operations. Next, the reader will learn why the probability of detection can be assumed equal to one for an array of one or more sensors, and how this differs from the probability of correct identification. After introducing the equations which drive the numerical analysis of the probability of identification, the reader will find a description of the measures of effectiveness used to analyze the optimal policy. Finally, the assumptions common to both models are discussed to prepare the reader for the models, whose introductions end the chapter.

An analysis of this complexity requires well defined notation. The reader will find definitions for all notation used, in order of appearance, given in the List of Symbols on page xv.

B. TERMINOLOGY

Before proceeding, it is necessary to specify the terminology to be used throughout this document. Additional guidance in decoding the numerous acronyms used is provided in the List of Abbreviations on page xiii.

1. Time Critical Target

A Time Critical Target (TCT) is any military vehicle which, from its standard tactics, can be expected to remain on the road system for a period not to exceed 30 minutes. As a result of this, actions taken to prosecute this type of target must be made expeditiously. TCTs include more than just Transporter-erector Launcher (TEL) units.

They may also be command vehicles, missile fuel trucks, missile loading trailers, and mobile Surface to Air Missile (SAM) units, among others.

2. "Steel Rattler" Unattended Ground Sensor

The "Steel Rattler" is a multi-component unattended ground sensor system with seismic, acoustic and infrared detection and identification capabilities designed and tested by Sandia National Laboratory in Albuquerque, New Mexico. The seismic/acoustic system first detects a target of interest and attempts to match its signature with a pre-loaded signature database. The time of detection, system position, and identification are sent via satellite link to a decision maker in the fusion center. If a positive identification is not possible, the seismic and acoustic systems "wake up" the infrared sensor which is positioned further along the expected route of travel. The infrared sensor transmits to the fusion center a still-photographic thermal image of the target of interest at its closest point of approach. From this image, a system operator identifies the target. If the seismic and acoustic systems make an identification, the infrared sensor remains dormant. Current units are designed to be hand placed; air deployable units are undergoing testing. Air deployable units will not have infrared sensors. The analysis in this thesis is based on the assumption that all sensors are identical "Steel Rattlers." More information about this sensor may be found in Appendix A.

3. Sensor Array

An array consists of one or more unattended ground sensors operating as a single entity. That is not to say that their information is shared. Each sensor operates independently, but the information returned from the sensors in the array is treated as a single set of data.

4. Expected Loss

To measure properly the effects of varying array characteristics, it is necessary to derive a utility function defining the relative value of all possible outcomes. Typically, one of the payoff values is set equal to one, and the other outcomes are specified in relative terms. For the purpose of this analysis, a *leaker* is given a value of two and a *false alarm* a value of one. That is, it is considered twice as detrimental to miss a TCT than it is to prosecute an innocent vehicle. The appendices include analyses for other relative values of leakers and false alarms.

5. Probability of Detection

The probability of detection, P_d, is the single incident Bernoulli trial probability of success associated with the likelihood that the sensor identifies the presence of a target within its detection range. It is important to note that a single sensor may have more than one opportunity to detect the presence of a target, depending on the target's speed and the sensor sample rate. This phenomenon is covered in greater detail later in this chapter.

6. Sensor Forecasts

In reality, the output of any sensor is a forecast. With this in mind, it is easy to see that the probability of identification, f_1, is the probability that the sensor correctly identifies the target in front of it as a TCT given that it is a TCT. Similarly, the probability of false-identification, f_0, is the probability that the sensor incorrectly identifies the target before it as a TCT when, in fact, it is not. These are values typically specified by the manufacturer of the sensor after operational testing .[Ref. 4]

C. PROBABILITY OF DETECTION

Modern acoustic, seismic and magnetic sensors sample at a fixed rate of t_s samples per second. The significance of this rate is that a TCT moving along a road may be investigated several times by the same sensor, thereby increasing the overall probability of detection. In fact, it can be shown that a "Steel Rattler" type of unattended ground sensor has a probability of detection approaching one against vehicles moving along roads at reasonable speeds.

To illustrate this effect, assume a single vehicle is traveling down a road with velocity v kph. Positioned along the road is an array of $n = 1$ sensor with maximum range R_{max} meters and sample rate t_s samples per second. The maximum *effective* range is determined by geometry to be r, and the sensor is positioned d_s meters perpendicular to the road, as shown in Figure 2.1, where $d_s < R_{max}$.

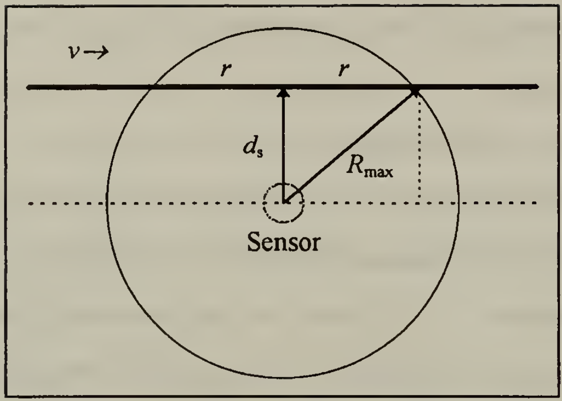

Figure 2.1 - Single Sensor Probability of Detection

From the figure above, it is evident that the vehicle will travel through $2r$ meters of sensor coverage for this particular sensor. Further, moving at speed v, the vehicle will travel

$$d_t = \frac{10}{36} \cdot \frac{v}{t_s} \text{ meters between sensor samples.}$$

Rearranging the above to solve for the number of samples available per sensor yields

$$s = \frac{2r}{d_t} = \frac{36}{10} \cdot \frac{2r \cdot t_s}{v} \text{ sensor samples.}$$

Therefore, as a function of the off-road distance, d_s, the number of samples available per sensor is given by

$$s(d_s) = \left\lfloor 7.2 \frac{t_s}{v} \sqrt{R_{max}^2 - d_s^2} \right\rfloor \qquad (2.1).$$

Note the use of the floor function to allow only integer values of $s(d_s)$.

Figure 2.2 shows the relationship between number of samples and off-road distance for a sensor with sampling rate $t_s = 0.2$ samples per second, and maximum range $R_{max} = 500$ meters, against a target moving at $v = 15$ kph.

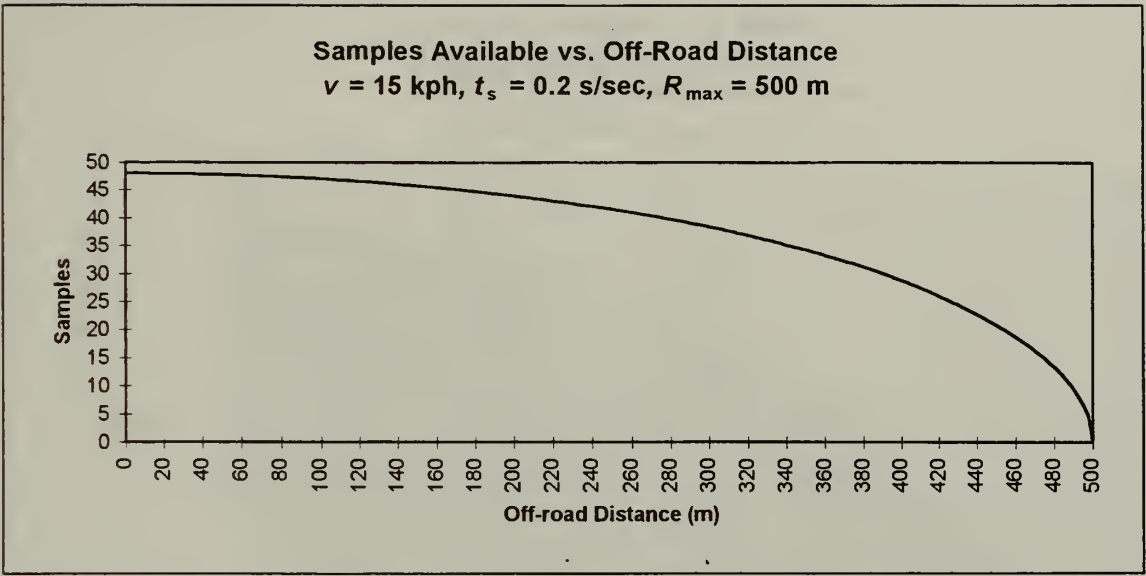

Figure 2.2 - Samples as Function of Off-road Distance

If the probability of detection for a single sensor sample is P_d, then the probability a vehicle is not detected on a single sample is $(1 - P_d)$. For a sensor sampling at rate t_s let P{detect | t_s} be the probability that it detects the passing vehicle. If the sensor is shown as located in Figure 2.1, then assuming independence,

$$P\{\text{detect} | t_s\} = 1 - (1 - P_d)^s, \quad (2.2)$$

where s is given by Equation (2.1). Values of P\{detect | t_s\} are shown in Table 2.1 for a sample size $s = 10$ with probabilities of detection ranging from 0.1 to 0.9. It is clear that even at a distance near the maximum sensor range, the overall probability of detection is high, even for low values of P_d.

Table 2.1 - Sensor $P\{\text{detect} \mid t_s\}$ for Sensor with Varying P_d

P_d	$P\{\text{detect} \mid t_s\}$
0.10	0.6513
0.20	0.8926
0.30	0.9718
0.40	0.9940
0.50	0.9990
0.60	0.9999
0.70	1.0000
0.80	1.0000
0.90	1.0000

Typically, TCTs do not travel at speeds greater than $v = 60$ kph. A graph illustrating the number of samples per sensor as a function of off-road distance and target speed is shown for distances of zero to R_{max} and speeds from 5 to 60 kph in Figure 2.3.

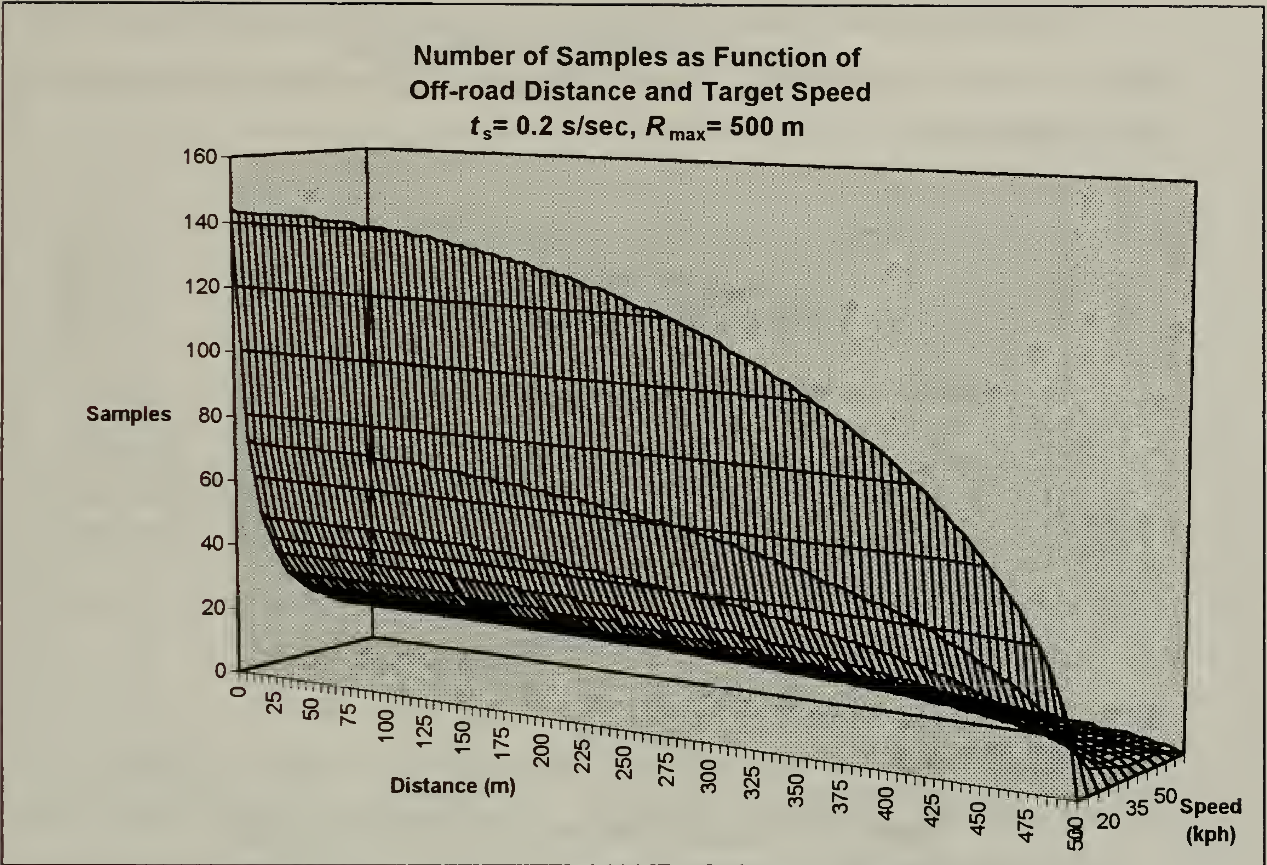

Figure 2.3 - Samples vs. Off-road Distance vs. Speed

The curve drops to zero at an off-road distance equal to the sensor's maximum range for obvious reasons. Clearly, in most cases there are more than enough samples for a single sensor to warrant the assumption that any target present is always detected. The exception is for extremely poor sensors, placed near the maximum range from the road against high-speed targets. But, since this analysis concentrates on the "Steel Rattler" type of sensor, this case does not apply.

Combining Equations (2.1) and (2.2) results in an expression for the probability of detection for a given sensor which accounts for several aspects of the problem. Sensor technology is represented by P_d, R_{max} and t_s. On the other hand, the operational characteristics of vehicle speed and off-road sensor distance are given by v and d_s. Equations (2.1) and (2.2) are combined to yield

$$P_{\text{detect}} = 1 - \left(1 - P_{\text{d}}\right)^{\left\lfloor 7.2\frac{t_s}{v}\sqrt{R_{\max}^2 - d_s^2} \right\rfloor}.$$

Detection probability versus off-road spacing is plotted for several values of P_d in Figure 2.4. In every case the P_{detect} drops to zero at $d_s = R_{max}$, as this reduces the effective sensor range to nil.

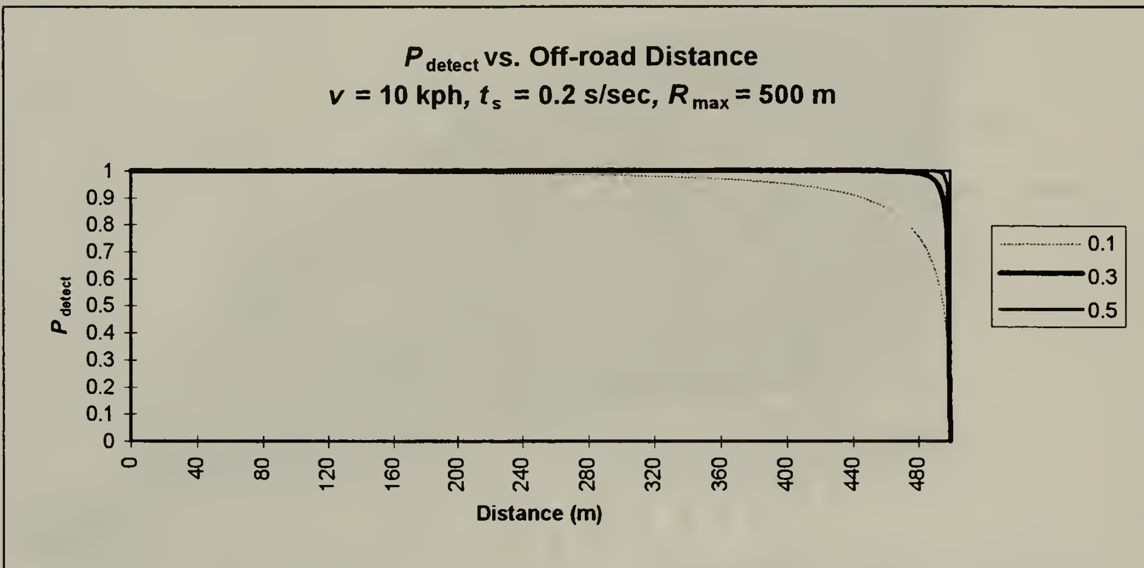

Figure 2.4 - P_{detect} as Function of Off-road Distance

Figure 2.5 provides an alternate way of analyzing this relationship. It shows the maximum allowable off-road distance, d_s, such that the probability of detection is at least 0.95, as a function of the single sample probability of detection, P_d.

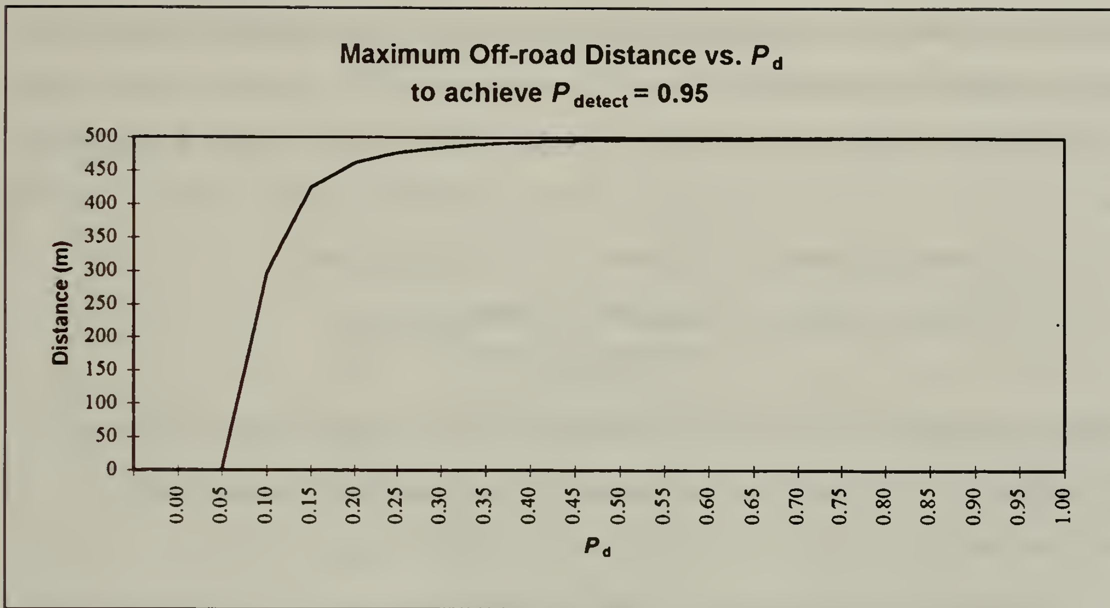

Figure 2.5 - Maximum Off-road Distance vs. P_d

The sensors illustrated in these examples pale in comparison to the "Steel Rattler" type of sensor, yet still produce exceptional probabilities of detection. The "Steel Rattler" is advertised to have a sample rate, t_s = 0.2 samples per second and a probability of detection, P_d = 0.95 [Ref. 5]. These values make the probability of target detection virtually one through its entire range. Hereafter, because of the sensor's sample rate and maximum range, a vehicle is assumed to be detected with probability one.

D. PROBABILITY OF IDENTIFICATION

In the previous section, the analysis of sensor detection indicates that a single sensor has nearly certain probability of detecting a vehicle moving along a road at a reasonable speed. The difficulty for the theater commander lies in determining when to commit scarce attack assets to prosecute a vehicle identified as a TCT based on data provided exclusively by ground sensors. The following discussion again assumes an array is comprised of n "Steel Rattler" type sensors, each of which relays a vehicle identification only once, at the closest point of approach, as it passes through the sensor's range. This identification is based on matching acoustic characteristics with an on-board signature file loaded prior to sensor deployment.

For each sensor in an array of size n, recall from Chapter II, Section B, Definition 6 that

$f_1 = P\{\text{Sensor indicates a TCT, given a TCT is present}\}$, and

$f_0 = P\{\text{Sensor indicates a TCT, given no TCT is present}\}$.

Throughout the thesis it is assumed that the sensor can discriminate between TCTs and other vehicles, so $f_1 > f_0$. Let P_x be the probability that an arbitrarily chosen vehicle traveling the road is a TCT (i.e. P_x is the fraction of vehicles that are TCTs).

The general formulation of probability of identification can be derived from Bayes' Rule and the Law of Total Probability. In the decision model described in Chapter III, the possible decision alternatives are to take action against a vehicle identified as a TCT, or take no action. The decision to take action is made when the theater commander has observed the number of sensors out of the entire array that indicate a TCT is present. Thus, the required identification probability to be used in the objective(Loss) function is $P\{\text{TCT} \mid k/n\}$, the probability that the detected vehicle is a TCT, given that exactly k out of n sensors in the array indicate it is. Assuming that the outputs of the array sensors are conditionally independent given the vehicle type, the decision probability is given by

$$P\{\text{TCT}|k/n\} = \frac{f_1^k(1-f_1)^{n-k}P_x}{f_1^k(1-f_1)^{n-k}P_x + f_0^k(1-f_0)^{n-k}(1-P_x)} \quad . \qquad (2.3)$$

To demonstrate the use of Equation (2.3), suppose there is an array of $n = 3$ sensors along a road with a TCT population of $P_x = 0.25$. The sensors are identical with $f_1 = 0.80$ and $f_0 = 0.15$. Table 2.2 shows the probability that a detected target is a TCT given k of n sensors indicate that it is for various values of k.

Table 2.2 - $P\{\text{TCT}\}$ Example Summary

k	$P\{\text{TCT}\|k/n\}$
0	0.0043
1	0.0896
2	0.6905
3	0.9806

The interested reader may turn to Appendix E for a more depth discussion of the derivation of Equation (2.3).

E. MEASURES OF EFFECTIVENESS

The theater commander must weigh all possible outcomes to arrive at his optimal policy. It is clear that this necessitates a common measure of effectiveness (MOE) used throughout the analyses. Possible candidate MOE's include the probability of an array correctly identifying a target and the loss incurred by acting incorrectly based on an array forecast. The former is maximized and the latter minimized.

Both of the above MOE's are used in this thesis because they are intertwined. The best array size is achieved by optimizing the probability of correct identification subject to constraints on the number of sensors, available intelligence, and potential array sites. Given the optimal size, the incurred loss MOE is used to determine the theater commander's decision policy. This relationship is made clear in Chapter III.

F. COMMON MODEL ASSUMPTIONS

Before the models for optimal sensor deployment are introduced, it is necessary to make some assumptions about the environment in which the problem is to be solved. The following assumptions hold true for both models used in the analysis of sensor configuration. First, there must be a finite number of sensors available to a theater commander for his area of responsibility. The employment of these sensors, however, is left to his discretion. If reality indicates that sensors are abundant, then the above constraint can be thought of as a logistical limit to the maximum number in theater at a given time. Next, it is imperative that the intelligence community be able to evaluate or estimate the fraction of vehicles in the area of responsibility believed to be TCTs. This assumption varies slightly between the two models, as shown in the following section. It is not imperative that the intelligence specialists give an exact percentage. Because the optimization is performed through a sensitivity analysis, an approximate range is sufficient and can be calculated from the enemy order of battle (OOB). Finally, as Model 2 will show, the optimization is best performed with a specific theater in mind. In this case, the number of arrays desired is an input value, and would not make sense otherwise. These simple assumptions set the stage for the two decision models below.

1. Model 1 Introduction

The first model is representative of the current, automatic "decision making" in which one good decision is assumed to be the answer to all problems. It is included in this thesis as a comparison to Model 2, which provides a response that changes with the specific theater, and is recommended. With the common assumptions above, the theater commander must choose n, the number of sensors in each array. Let N be the number of sensors available and let A be the number of candidate array sites. Then, $An \leq N$. The value of n may come from some tactical publication, a rule of thumb, or a hunch. In Model 1, n is chosen mathematically by dividing the number of sensors available by the number of array sites, and rounding down to the next lowest integer. Thus, $n = \left\lfloor \frac{N}{A} \right\rfloor$. Since all of the arrays are of equal size, the optimal solution may be found directly by arithmetic means. A significant drawback of this method is that sensors for additional arrays may not be available later in the conflict. Summarizing, in Model 1 the problem is to specify locations in the theater commander's AOR at which arrays should be placed. Array size is the direct result of distributing the available sensors equally among the locations.

2. Model 2 Introduction

The second model presents a more pragmatic, yet flexible approach to the sensor array problem. It uses the power of information gathered by intelligence analysts on the best candidate array sites for a given theater. In addition to a fixed N, it is assumed that a reasonable number of prospective sites has been chosen by professionals in the intelligence or special operations communities. In this case, the value $n = \left\lfloor \frac{N}{A} \right\rfloor$ may not represent the optimal array size for a given location. In fact, roads with differing fractions of TCT traffic are expected to be best covered by arrays of differing size. Since the array positions are chosen in advance, it is further assumed that some estimate of TCT traffic can be made for each road to be seeded. Let i be one of the roads to be seeded with an array. Further, let $P_x(i)$ be the fraction of traffic along road i comprised of TCTs. Then there is a specific n_i* corresponding to each $P_x(i)$. For each of these A arrays there will be an optimal policy which dictates when the theater commander should commit an attack asset to a target.

Figure 2.6 illustrates the general problem flow and the solutions obtained. Chapter II introduced material above the dashed line, including model inputs and primary outputs of n_i* and k_i*. Inputs include the maximum number of sensors available, N, the number of candidate array sites, A, and the estimated TCT fraction of the population, $P_x(i)$. Chapter III demonstrates solutions to the problems displayed below the dashed line. A brief sojourn above the line gives definitive solutions for the n_i* and k_i* introduced in this chapter. Next, using the theater commander's maximum desired reporting time for the entire array, t_R, an inter-sensor spacing is determined, d. As an aside, these values may be combined to evaluate P_c, the probability that the entire array is compromised given a single sensor is found. With the array size, policy and spacing found, the theater commander must now weight the array attributes of reporting, compromise, and countermeasures in accordance with his own view of the tactical situation. From these inputs the optimal array placement geometry may be found, concluding the problem.

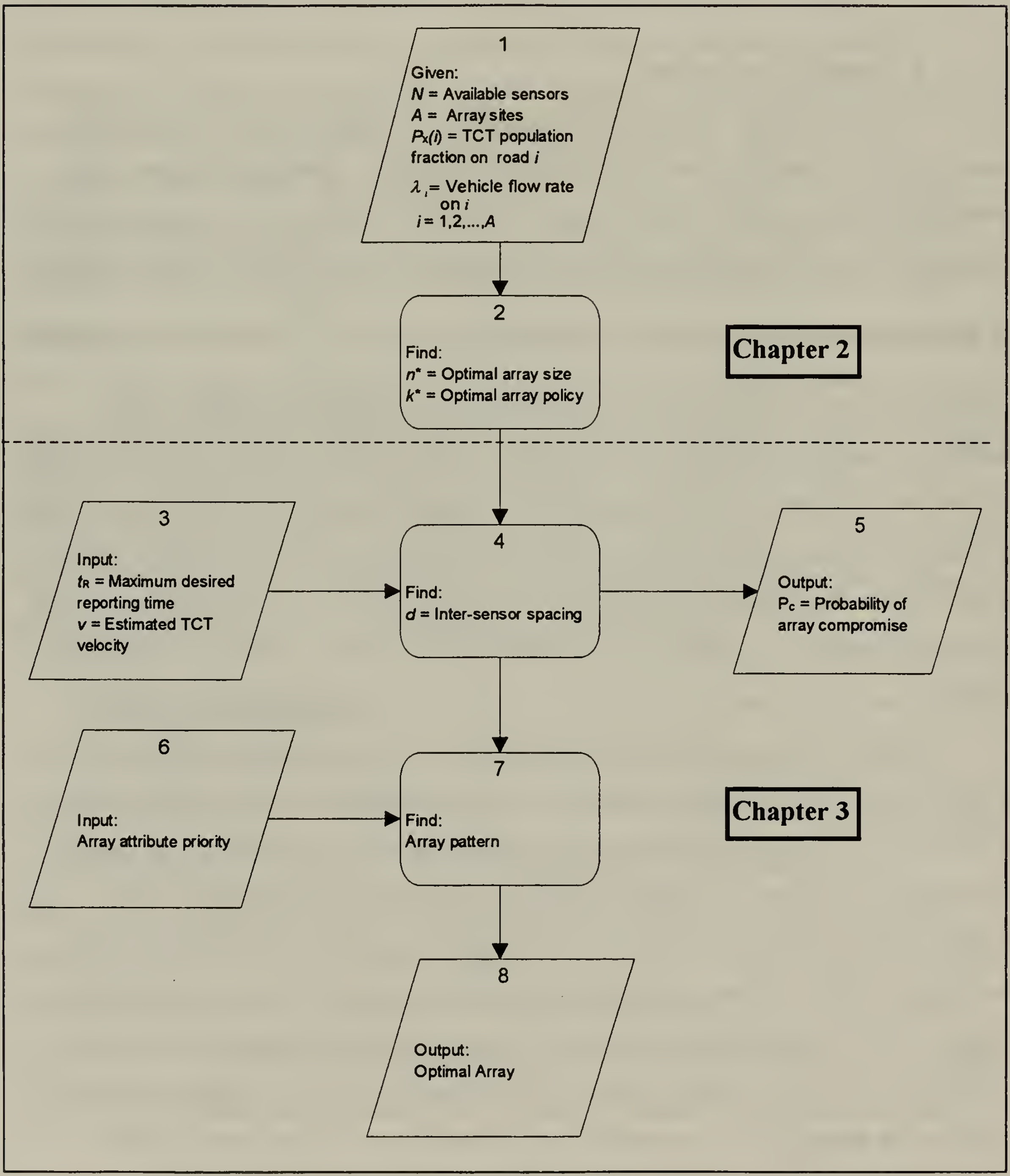

Figure 2.6 - Problem Flow

III. ARRAY CONFIGURATION

A. INTRODUCTION

For reasons both tactical and financial, much emphasis is placed on the efficiency and reliability of the information reported by the ground sensors. With a finite number of sensors available, it is imperative that the arrays be configured so as to produce the most reliable forecast to the theater commander at the minimum unit cost.

Before continuing, it is necessary to introduce the problem to be solved by the models. In some specific theater, there is a network of roads along which time critical traffic, as well as normal civilian traffic, is known to travel. The fraction of vehicular traffic that is TCT is P_x, and the fraction along a specific road i of interest is $P_x(i)$, $i = 1,2,\ldots,A$. Further, the theater commander has been allocated N unattended ground sensors to locate and identify the TCT traffic, to be deployed at his discretion. Let these sensors have performance characteristics of $f_1 = 0.80$ and $f_0 = 0.15$, as described previously. Further, it is assumed that hostilities have erupted, so the "cost" of mis-identifying an actual TCT is $r_1 = 2$ and the "cost" of a false alarm is $r_2 = 1$. That is, a leaker is twice as costly as a false alarm. . Using this model, the following discussion details methods for determining the optimal array size, spacing and deployment pattern for the two models specified in Chapter II, Section F.

B. ARRAY SIZE AND OPERATING POLICY

1. Model 1

The array size in Model 1 is the same for all locations and is predetermined to be n. This n is simply a function of the number of candidate array sites, A, and is given by $n = \left\lfloor \frac{N}{A} \right\rfloor$, where N is the number of sensors available. With the array size fixed, the sensitivity analysis consists of finding the optimal policy as a function of the overall TCT population in the theater. Let k_i be the minimum number of sensors indicating a vehicle is a TCT on road i required for the theater commander to commit an attack asset. Then, the

k_i or more of n policy defined in the previous chapter, Section D, may be found by solving the nonlinear program for each road, i:

$$\begin{aligned}
&\textit{Minimize} \quad l_i(n,k_i) \\
&s.t. \\
&1 \le k_i \le n \\
&P_x = \text{constant} \\
&k_i \quad \textit{Integer} \\
&i = 1,2,\ldots,A
\end{aligned}$$

where $l_i(n, k_i)$ is the expected loss function derived in Appendix F, and shown below as Equation (3.1). The binomial distribution function given by $\sum_{j=0}^{k_i} \binom{n}{j} f_0^j (1-f_0)^{n-j}$ is abbreviated as $B(k_i, n, f_0)$.

$$l_i(n,k_i) = r_1\, B(k_i - 1, n, f_1) P_x(i) + r_2 \left[1 - B(k_i - 1, n, f_0)\right](1 - P_x(i)). \quad (3.1)$$

Optimum k_i* values for varying P_x and array sizes, n, are shown in Table 3.1 for arrays up to size $n =$ 10.

Table 3.1 - Optimum Model 1 Policies

P_x	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
0.10	1	2	2	3	3	4	4	5	5	6
0.20	1	2	2	3	3	4	4	4	5	5
0.30	1	1	2	2	3	3	4	4	5	5
0.40	1	1	2	2	3	3	4	4	5	5
0.50	1	1	2	2	3	3	4	4	5	5
0.60	1	1	2	2	2	3	3	4	4	5
0.70	1	1	1	2	2	3	3	4	4	5
0.80	1	1	1	2	2	3	3	4	4	4
0.90	1	1	1	1	2	2	2	3	3	4

For example, if the theater commander is allotted $N = 12$ UGS for his AOR consisting of $A = 3$ array sites, then he will deploy three arrays of size $n = 4$. His intelligence team estimates the fraction of vehicles which are TCT's on the three roads are $P_x(1) = 0.30$, $P_x(2) = 0.10$ and $P_x(3) = 0.20$. Therefore, he knows from Table 3.1 that he should prosecute the target only if $k_1^* = 2$, $k_2^* = 3$ and $k_3^* = 3$ or more of the three sensors in a given array identify the target as a TCT. These results are summarized in Table 3.2.

Table 3.2 - Model 1 Example Results

Array	$P_x(i)$	n	k_i^*
I	0.30	4	2
II	0.10	4	3
III	0.20	4	3

The commander knows that this policy will minimize losses due to leakers and false alarms, thereby optimizing his asset allocation.

2. Model 2

Array size is the essence of the problem for Model 2. Remember that each array potentially has a different number of sensors based on the fraction of TCT's on the specified road, $P_x(i)$. Let n_i be the number of sensors used at location i, and let λ_i be the traffic flow rate along road i measured in vehicles per hour. In this case, the only restrictions on the n_i are that they be integers, that $\sum n_i \le N$, and that each prospective location has a deployed array of at least one. This leads directly to an optimization of the form:

$$
\begin{aligned}
& Minimize \sum_{i=1}^{A} \lambda_i l_i(n_i, k_i) \\
& s.t. \\
& \sum_{i=1}^{A} n_i \le N \\
& n_j \ge 1, \quad \forall j = \{1, 2, \ldots, A\} \\
& 1 \le k_j \le n_j, \quad \forall j = \{1, 2, \ldots, A\} \\
& n_j, k_j \quad Integer
\end{aligned}
$$

where l_i is the optimal expected loss for array i, shown in Equation (3.2) and derived in Appendix F.

$$l_i(n_i, k_i) = r_1 B(k_i - 1, n_i, f_1) P_x(i) + r_2 \left[1 - B(k_i - 1, n_i, f_0)\right](1 - P_x(i)) \quad . \tag{3.2}$$

For example, assume the same theater commander is again allotted $N = 12$ UGS for his AOR, which has $A = 3$ candidate array sites. His intelligence team estimates the fraction of vehicles which are TCT's on these three roads are $P_x(1) = 0.30$, $P_x(2) = 0.10$ and $P_x(3) = 0.20$. Additionally, the flow rates for these roads are estimated at $\lambda_i = 1$ vehicle per hour, for all i. After solving the nonlinear integer program outlined above, the optimal array sizes are $n_1^* = 3$, $n_2^* = 4$ and $n_3^* = 5$. The optimal k_i^*, given these n_i^* are $k_1^* = 2$, $k_2^* = 3$ and $k_3^* = 3$. These results are summarized in Table 3.3.

Table 3.3 - Model 2 Example Results

Array	$P_x(i)$	n_i^*	k_i^*
I	0.30	3	2
II	0.10	4	3
III	0.20	5	3

Again, the commander knows that losses due to leakers and false alarms will be minimized if the k_i^* or more of n_i^* policy indicated is adhered to for a particular array. Given that

k_i* or more do indicate a TCT, he should prosecute the target, now identified as a TCT, with an available asset.

A comparison of the results reveals the advantages of Model 2 over Model 1. Continuing with the assumption that the same three roads were seeded in the given theater of operations gives the results summarized in Table 3.4. Model 1 determines a $k_1^* = 2$, $k_2^* = 3$ and $k_3^* = 3$ or more of $n = 4$ policy, and Model 2 determines the policies summarized in Table 3.3. In both cases the corresponding road populations, $P_x(i)$, are used for the loss function.

Table 3.4 - Model Comparison Summary

	Expected Loss on Road			
	1	**2**	**3**	**Total**
Model 1	0.0930	0.0469	0.0819	**0.2218**
Model 2	0.1049	0.0469	0.0445	**0.1963**

Clearly, even for this small example, Model 2 shows superior performance at a cost of twelve sensors for the theater. As the number of candidate sites and sensors increases, so do the savings in manpower for deployment, and sensor equipment.

C. ARRAY SPACING

With the array size and identification policy determined, the next logical question to answer is how far apart should the sensors of a particular array be spaced, and how far from the road should they be. The second half of the question is more subjective, and therefore will be addressed first. In the case of air dropped sensors, it is likely that considerable error will be associated with the deployment and free-fall of the individual units. In this case, it seems smartest to aim at a position half of the maximum radius away from the road. Location error in either direction will then still allow the sensor to function with some or all of its capability. Sensors placed by special operations units are positioned with a greater degree of accuracy, and will always be within the range of the sensor's capabilities. Therefore, the off-road distance of the sensor should be left to the discretion of the insertion team leader, who should receive some training in placement. In general, the distance should be as close to the road as is operationally possible, without risking compromise of the sensor or the insertion team.

Inter-sensor spacing is a function of several different factors. Generally, the sensors need to be close enough to each other that the theater commander can consider the report cycle of all sensors in an array as a single event. On the other hand, they should be spaced far enough apart to minimize the likelihood that the entire array is compromised should a single sensor be discovered. Clearly, the probability that the entire array is compromised increases with decreasing inter-sensor distance. The goal is to find a middle ground acceptable to the theater commander.

To analyze this relationship, assume the hostile force has found a sensor and will conduct a random search of the surrounding area for some time, t, minutes. Then, assuming that m individuals each search randomly and uniformly at a rate of S m^2/second, these individuals cover a total area of $A_s = Smt$ m^2 during the search time. Using a standard area search model, the area to be searched is actually a circle with a radius equal to the distance between the farthest two sensors. This data is obtained by the enemy observing American standard operating procedures during the conflict, and correctly estimating n. An upper bound on this radius is found by assuming the sensors are arrayed in the line pattern configuration and that the sensor initially found is at the end of the line. This value is easily seen to be radius = $d(n\text{-}1)$. Let P_c be the probability an additional array sensor is compromised, given that a single sensor has been found. Then, from Random Search Theory, the probability the hostile force finds one additional sensor in time t is given by

$$P_C(d) = 1 - e^{-\frac{60Smt}{\pi(d(n-1))^2}}. \qquad \text{[Ref. 6].}$$

This probability of compromise, P_c, is shown on the left hand curve in Figure 3.1 for varying inter-sensor range, d. Other parameters used in this illustration are:

- $n = 4$ sensors
- $S = 24$ m^2/ second
- $m = 4$ searchers
- $t = 240$ minutes

For example, if four sensors are spaced at an inter-sensor distance of $d = 300$ meters, and the search effort is as indicated as above, then the probability that one additional sensor is discovered in the allotted search time is:

$$P_c(300) = 1 - e^{-\frac{60\cdot 24\cdot 4\cdot 240}{\pi(300(4-1))^2}}$$
$$= 1 - e^{-0.54325}$$
$$= 0.419$$

Therefore, there is a 41.9% chance that the searching party will find one additional sensor in $t = 240$ minutes.

At the same time, the theater commander is awaiting the full report of his array. This time increases linearly with increasing inter-sensor distance, d, and is shown as the straight line in Figure 3.1. This figure assumes that the TCT is traveling at a constant speed, $V_{TCT} = 20$ kph.

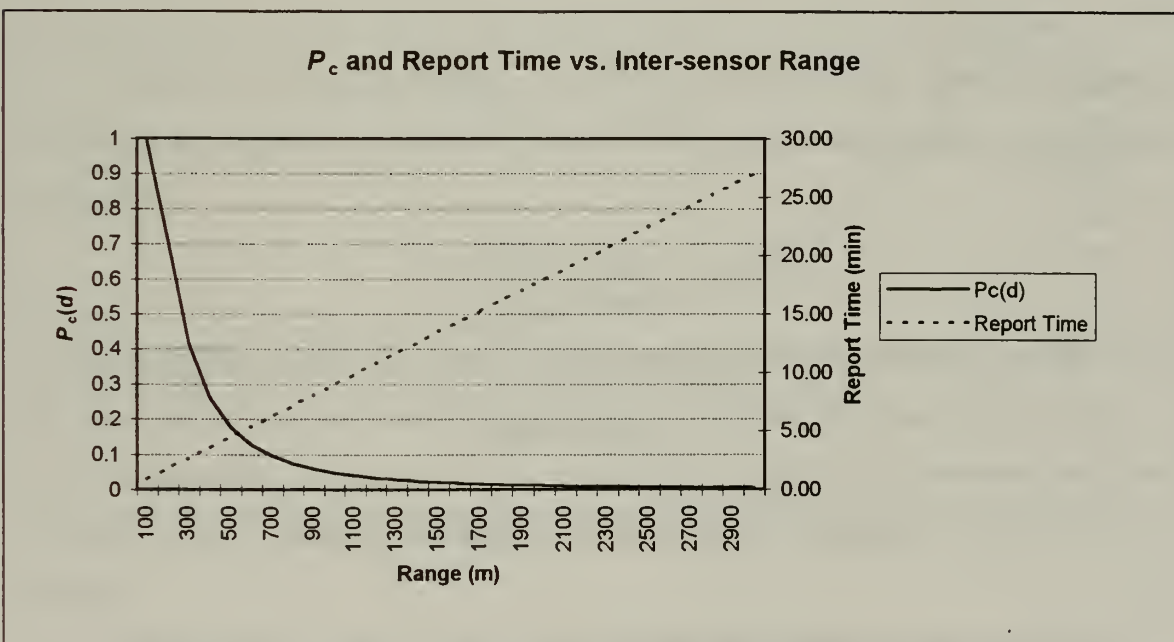

Figure 3.1 - Effect of Inter-sensor Range on P_c and Report Time

The graph illustrates two distinct features. The left hand curve depicts the decreasing probability that the search team compromises an additional sensor in the allotted time as the sensors become spaced further apart. The linear curve shows the increase in reporting time for the entire array as this inter-sensor distance increases. These two relations may be combined to obtain the probability of compromising one or more additional sensors as a function of array reporting time, given as Equation (3.3). It is important to remember that the values shown in Figure 3.1 are obtained using the line pattern, and therefore represent an upper bound.

$$P_c(t_R) = 1 - e^{-\frac{60Smt}{\pi\left(\frac{25}{3}vt_R\right)^2}}, \qquad (3.3)$$

where t_R is the reporting time for the entire array. A plot of P_c for varying t_R is shown in Figure 3.2 for the parameters specified above.

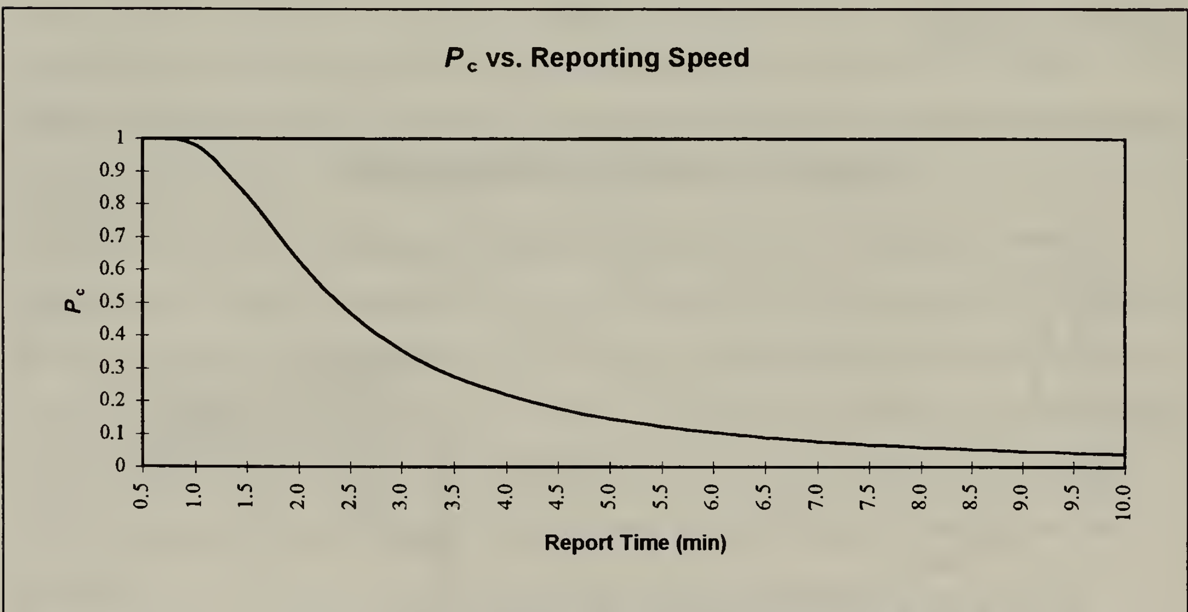

Figure 3.2 - P_c as Function of Reporting Speed

It is important to remember that most of the factors involved in this curve are beyond the control of the theater commander. Placement errors from air drops or hand emplacement can influence the inter-sensor distance, and the opposing search team's capabilities, size, and available time are unknowns. These formulae above simply serve as a general reference as to the overall mission capability of the system using estimated parameters. Table 3.5 gives the inter-sensor range, *d*, for varying array sizes, *n*, and maximum reporting times, t_R, based on a TCT with estimated speed $v = 20$ kph. Tables for other common TCT speeds are compiled in Appendix D.

Table 3.5 - Inter-sensor Distance (meters) for TCT Speed $v = 20$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	333	167	111	83	67	56	48	42	37
2	667	333	222	167	133	111	95	83	74
3	1000	500	333	250	200	167	143	125	111
4	1333	667	444	333	267	222	190	167	148
5	1667	833	556	417	333	278	238	208	185
6	2000	1000	667	500	400	333	286	250	222
7	2333	1167	778	583	467	389	333	292	259
8	2667	1333	889	667	533	444	381	333	296
9	3000	1500	1000	750	600	500	429	375	333
10	3333	1667	1111	833	667	556	476	417	370

Table 3.5 is based on Equation (3.4) which gives inter-sensor distance in meters as a function of array size, estimated TCT velocity in kph, and maximum array reporting time in minutes,

$$d = \frac{1000}{60} \cdot \frac{v \cdot t_R}{(n-1)} \quad . \qquad (3.4)$$

It is important to note that the number of sensors in the array for the look-up tables and for Equation (3.4) represent only the sensors not making simultaneous reports. In the case where *w* sensors report to the theater commander virtually simultaneously, only one of the *w* is used in computing the array size. This is described further in the following section.

D. ARRAY PLACEMENT PATTERNS

The final part of this chapter covers the actual geometry of the sensor array as viewed from above. Each of the four proposed patterns has strengths and weaknesses when evaluated in the areas of reporting time, compromise and countermeasures as described below.

First, reporting time refers to the speed, in minutes, at which the entire array can deliver *n* reports to the theater commander. Next, compromise is the probability that additional array sensors are found given that one has been discovered. Finally, countermeasures refers to the likelihood that more than one sensor may be affected if some form of jamming is used in its proximity. Also included in this category are environmental effects which may hamper sensor performance, such as wind gusts or a falling branch.

When seeding a road, sensors spaced equidistant from each other fall into four proposed patterns: the line, the cross-hatch, triples, and the goal post. These are summarized and illustrated in Figures 3.3 through 3.6, below. For each figure, d represents the inter-sensor spacing and d_s the off-road distance. Larger arrays than those shown can be built by adding two of the above simple pattern units, or by continuing the obvious pattern.

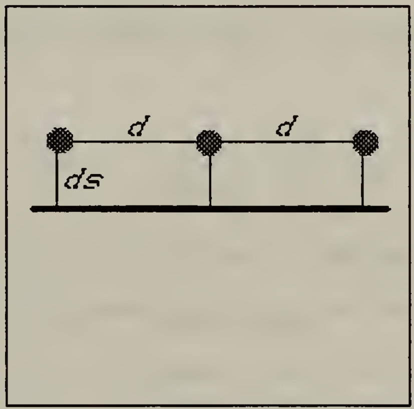

Figure 3.3 - Line Pattern

The line pattern is the simplest of the building blocks used for array placement. Sensors are placed on one side of the road with a constant inter-sensor distance, d meters, and an off-road distance, d_s meters. This pattern is also the easiest to lay, either by hand or by air drop.

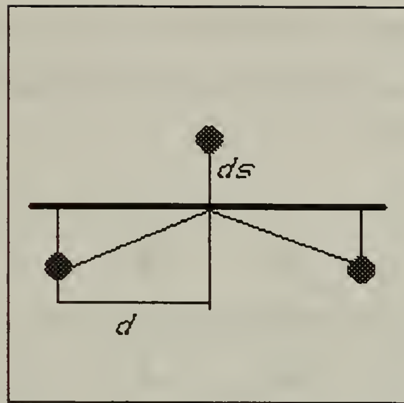

Figure 3.4 - Cross-Hatch Pattern

The cross-hatch pattern alternates sensors on either side of the road, again with an effective along-road spacing of *d*. This pattern would be extremely difficult to deploy from the air. If it became necessary to do so, the aircraft would have to fly along either side of the road on separate runs, spacing the sensors a distance of *2d* meters apart. Again, it would be unlikely that the configuration between the two rows would be properly aligned with this technique. With this in mind, it is recommended that this pattern be reserved for SOF deployment.

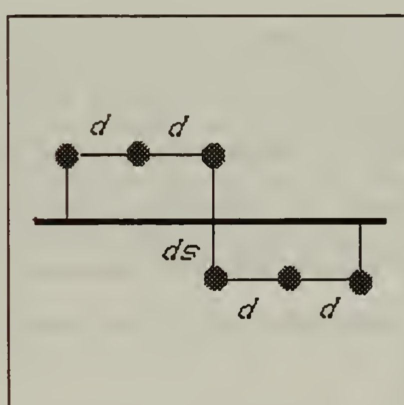

Figure 3.5 - Triples Pattern

The triples pattern combines the line and the cross-hatch patterns. In fact, this pattern could also be produced with quadruples, or more. Similar deployment problems as with the cross-hatch are evident here, also. An advantage of the triples pattern is that every *2d* meters of TCT transit there is a simultaneous report from two sensors.

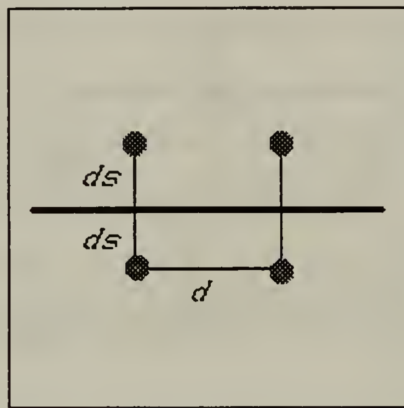

Figure 3.6 - Goal Post Pattern

Finally, the goal post pattern is a combination of two line patterns, one on either side of the road. It can easily be deployed by both land and air forces. The greatest advantage of the goal post, is that the theater commander receives two sensor reports virtually simultaneously every *d* meters. The weakness is that any environmental effects or countermeasures affecting one sensor will probably also negate its counterpart across the road.

The performance of each of the patterns described above is scored on a scale of one to four and their relationships are illustrated on a policy diagram. In each case, a higher score is desired. Operationally, scores are obtained from the tactical experience of the theater commander. As an example, the author has provided scores for the overall performance of the sensor patterns when evaluated for reporting speed, likelihood of compromise, and susceptibility to countermeasures or environmental factors. These values are summarized in Table 3.6.

Table 3.6 - Placement Pattern Summary

Pattern	Reporting s_1	Compromise s_2	Ctr. Measures/ Environment s_3	$\sum_{i=1}^{3} w_i s_i$ $w_i = 1/3\ \forall i$
Line	1	1	3	1.67
Cross-hatch	1	4	4	3.00
Triples	3	3	2	2.67
Goal Post	4	2	1	2.33

The triples and goal post patterns score high in the reporting category because two sensors relay information virtually simultaneously at most every $2d$ meters. This significantly speeds up the reporting time of the array. On the other hand, this arrangement could lead to greater susceptibility to countermeasures or compromise, and so they score lower in this attribute. The cross-hatch pattern scores high against both countermeasures and compromise because the inter-sensor distance on one side of the road is $2d$, or twice the actual inter-sensor distance. However, a report is only received every d/v time units rather than 2 reports every d/v time units, and therefore a lower reporting score.

The last column represents the relative values of the patterns if all three attributes are equally weighted, an unlikely occurrence for any decision maker. Therefore, a sensitivity analysis of the weights was performed using the ranks in Table 3.6. The policy space of the relative weights of the different categories is shown in Figure 3.7

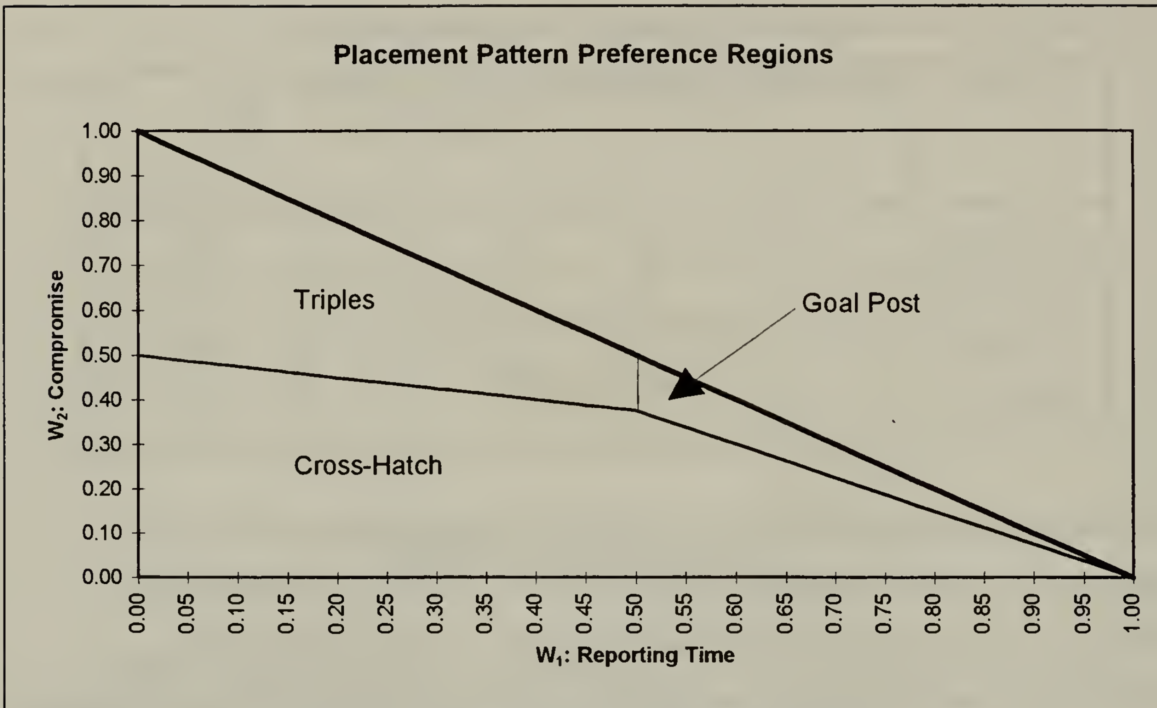

Figure 3.7 - Placement Preference Regions

where w_1 and w_2 represent the relative weights associated with reporting time and compromise. The sum of the weights on reporting time, compromise, and countermeasures is equal to one, or

$$\sum_{i=1}^{3} w_i = 1.$$

The absence of the line pattern altogether is due to its domination by the cross-hatch pattern in the feasible region. For theater commanders who weigh reporting time heavily, cross-hatch is the pattern of choice in most cases. On the other hand, if compromise is paramount and reporting time of little value, then triples should be chosen. If all attributes are equally weighted, then the diagram indicates cross-hatch should be chosen as was indicated in Table 3.6.

E. CONCLUSIONS

This section is best summarized by returning to Figure 2.6, reprinted below as Figure 3.8. This flow diagram may be used as a checklist for the theater commander when deploying his forces to minimize the TCT threat.

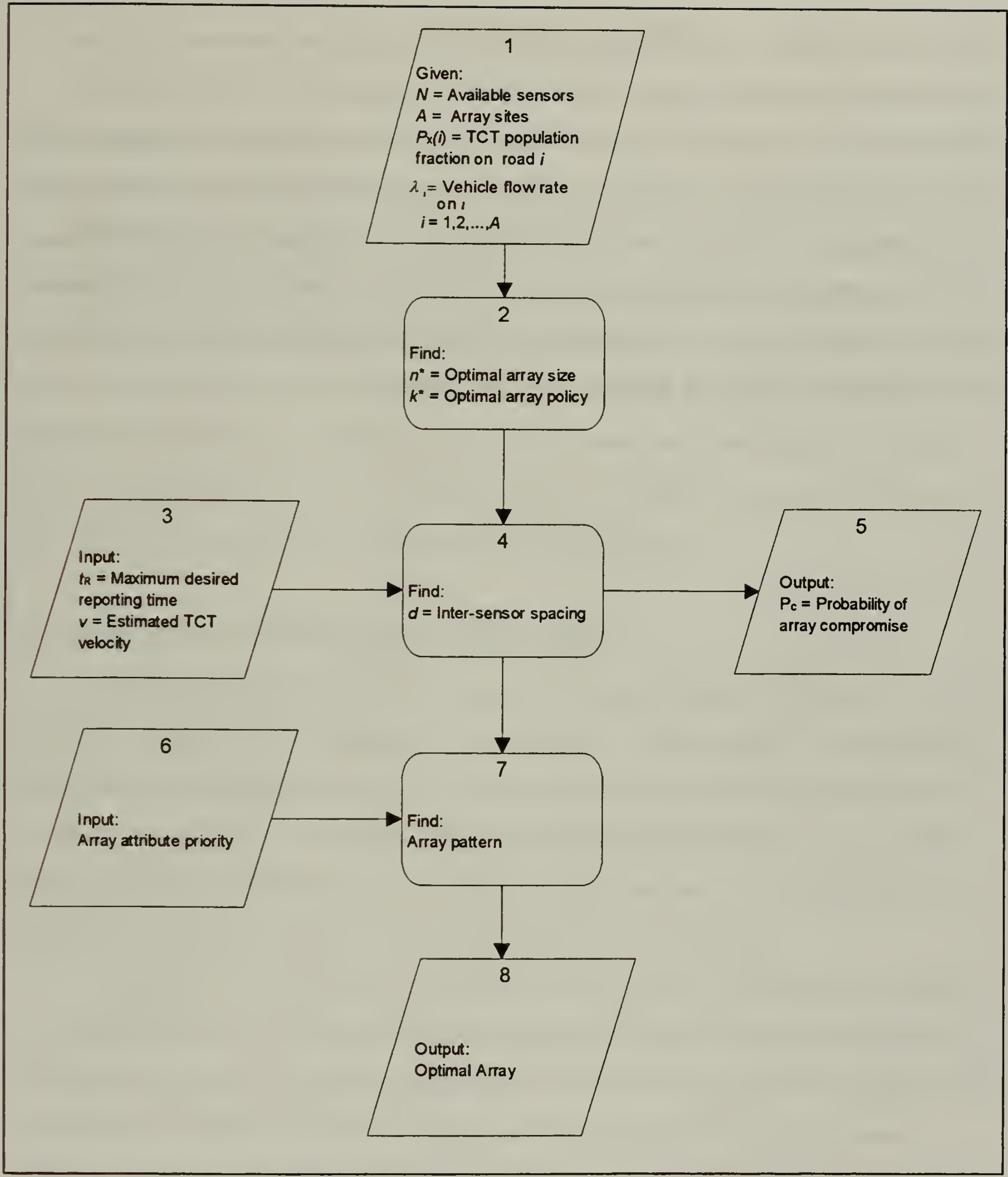

Figure 3.8 - Array Deployment Checklist

The inputs include the array stockpile size, the candidate array sites, vehicle flow rates, and corresponding TCT population fractions for those sites. From the nonlinear program specified in Section B an optimal array size and k_i* are obtained. Next, the theater commander must decide on his maximum array reporting time and estimated TCT velocity along the seeded road. These estimates, along with the array size above will yield the recommended inter-sensor distance when applied to Equation (3.4). With reasonable estimates of enemy capability, the probability of compromise may also be obtained at this point using Equation (3.3). Finally, the decision maker must prioritize the relative weights of reporting time, compromise, and countermeasures. These weights, when applied to Figure 3.7, return the optimal array pattern. Therefore, the complicated task of choosing the optimal array for a given set of roads in a theater of operations has been reduced to three simple decisions for the theater commander.

IV. ARRAY LOCATION

A. INTRODUCTION

This chapter addresses the specific location of the array along a chosen road to maximize the array's strengths and minimize the chance of compromise in accordance with the theater commander's mission priorities. It is assumed that the intelligence community is responsible for choosing the particular roads to be seeded based on TCT traffic, mission criticality, and probability of mission success. The specific areas of placement considered are straight road segments, intersections, and geographic choke points. Each is described, then evaluated by sensitivity analysis to determine which best suits the theater commander's assessment of the tactical picture.

B. GENERAL PLACEMENT CONCERNS

Before the different possible locations are measured against one another, it is first necessary to determine the attributes most important to mission success. These attributes, placement and compromise, information, and tactical potential, are then used to evaluate the relative strength of each location. The performance of arrays in each of the possible locations described below is scored on a scale of one to four and their relationships are illustrated in a policy diagram. In each case, a higher score is desired.

Placement involves the ease with which the array may be deployed by SOF or by air. Placement not only includes the physical difficulty in laying the sensors, but also the likelihood that the insertion team is discovered before the array is completely deployed and camouflaged. Placement is grouped with compromise because they share the same strengths and weaknesses, thereby making their scores equal and uninteresting. That said, compromise, unlike the array spacing analysis, addresses the probability that the first sensor of an array is discovered by random sweeps conducted by hostile forces. This is based on the assumption that the opponent knows the United States is using ground sensors for cueing strike assets, and is sweeping areas deemed most likely to be harboring arrays. Information is obtained from the raw data produced by the array. This includes both positive and negative information, which are defined as the presence of a TCT, and the lack of TCT contact, respectively. The tactical potential score is based on the clarity of the information received by the theater commander from the array. Specifically, the information attribute addresses the ability to transfer data to an airborne strike asset. Locations which allow an airborne platform to easily locate and positively correlate a vehicle with the array output are high on the scoring scale. Those that provide confusing or ambiguous information score low. The three attributes are weighted in relative importance by the theater commander and the optimal array location is read from the policy graph later in this chapter.

C. STRAIGHT ROAD SEGMENT

Locating an array along a straight road segment is probably the simplest, and most practical insertion technique. Since straight roads are far more abundant than choke points or intersections, it is a simple matter for a trained unit to choose an easily accessible section of road, deploy the array and withdraw. Similarly, air-dropped arrays are equally effective along any section of a straight road, and the exact location may be chosen to minimize the possibility of action by hostile forces, both against the array and the deploying aircraft. Because road segments are so abundant, forces sweeping for arrays will have little success. It would be a difficult task, without some kind of cueing to isolate a particular section of straight road along which to conduct a search for an array. For this reason, it is hypothesized that enemy search forces will concentrate on sweeping choke points and intersections rather than on open roads. The geometry of road segments precludes excessive traffic, and the specific volume of traffic is a function of the road chosen, not the segment. Finally, road segments provide a clear tactical picture in that there is only one entry and one egress from a road segment. Therefore, a cued air asset should easily locate and visually identify a TCT traveling down a straight road. The scores for the straight road segment for the above attributes are shown in Table 4.1.

Table 4.1 - Straight Road Segment

Attribute	Score s_1
Placement & Compromise	3
Information	1
Tactical Potential	2

These scores are used in the sensitivity analysis against the other array locations.

D. INTERSECTION

Intersections naturally attract significant attention due to the seemingly endless possibilities they provide. A TCT is easily tracked until it reaches an intersection. Then, unless each of the exiting segments contains an array, it could simply vanish from the tactical picture of the theater commander. Similarly, if two arrays on opposite sides of an intersection gain contact in a reasonable time increment, who can positively state that there is only one TCT operating in the area? Perhaps the original TCT turned, and a second unit is passing the other sensor. These problems plague road intersections and may not be easily answered. The only definite solution is the use of a SOF team at the intersection to visually identify each TCT as it passes.

The general business of an intersection automatically makes array placement by SOF team more difficult. Although not all intersections are busy, they are by nature more traveled than straight road segments. Following the above hypothesis that enemy sweeping action will be concentrated at intersections and choke points, makes arrays placed at intersections more subject to compromise. Further, the likelihood that a given intersection is searched grows with the relative importance of that intersection as a military transit hub. Obviously, intersections near to forward assembly areas will be swept regularly. The real strength of intersections is the volume of information they produce. The sheer amount of traffic flowing through a busy intersection provides an excellent sample of vehicle population of all types, TCT and otherwise. A seeded intersection with no TCT contact provides as much, if not more, information as a positive contact along a straight road. The negative information associated with the intersection implies not only that the intersection sees no TCT traffic, but also that the road segments adjacent are not used by TCT's. This can significantly reduce the overall search area for other assets and can help determine array locations for future array deployment sites. Tactically an intersection provides little aid in the prosecution of TCT's by air assets. A TCT passing through an intersection is generally lost until it passes a more specific identification point, such as a choke point or an array along a straight road segment. Array output is generally too vague to determine which branch the TCT took when exiting the intersection. Table 4.2 lists the scores for an intersection as a placement location.

Table 4.2 - Intersection

Attribute	Score s_2
Placement & Compromise	2
Information	3
Tactical Potential	1

E. GEOGRAPHIC CHOKE POINT

Geographic choke points share the best and the worst characteristics of the above two locations. Placement is difficult due to the very nature of the choke point. Entry and egress to the area may be difficult, and it may be well patrolled because of its significance. Additionally, since the opposing forces must also realize this area is a choke point, it is a very likely candidate for sweeps, making the risk of compromise greater. With less area to search, arrays in these areas are at high risk. Depending on the particular choke point, information provided may be quite plentiful. If the area is one of few allowing passage between hostile depots and their forward staging areas, much information will be available. Similarly, a bridge or causeway frequently used to move military vehicles is a good target.

Finally, the tactical use of a geographic choke point is incomparable. A targets moving into a choke point is restricted in movement and may be waited for as it egresses. This would allow an easy transition from ground information to air. The attribute scores for the geographic choke points are given in Table 4.3.

Table 4.3 - Geographic Choke Point

Attribute	Score s_3
Placement & Compromise	1
Information	2
Tactical Potential	3

F. SENSITIVITY ANALYSIS

A summary of the overall value of the different location areas when evaluated for placement and compromise, information, and tactical potential is given in Table 4.4. The last column indicates the value of the location areas with all attributes weighted equally. Note that each area totals to a value of two. This implies that each of the areas has strengths and weaknesses in the attributes evaluated, and that the most suitable location depends greatly on the preferences of the theater commander.

Table 4.4 - Location Area Summary

Location Area	Information s_1	Tactical Potential s_2	Placement & Compromise s_3	$\sum_{i=1}^{3} w_i s_i$ $w_i = 1/3 \; \forall i$
Straight Road	1	2	3	2
Intersection	3	1	2	2
Choke Point	2	3	1	2

The policy space of the relative weights of the different categories is shown in Figure 4.1, where w_1 and w_2 represent the relative weights associated with information and tactical potential. The sum of the weights on information, tactical potential, and placement and compromise is equal to one, or $\sum_{i=1}^{3} w_i = 1$.

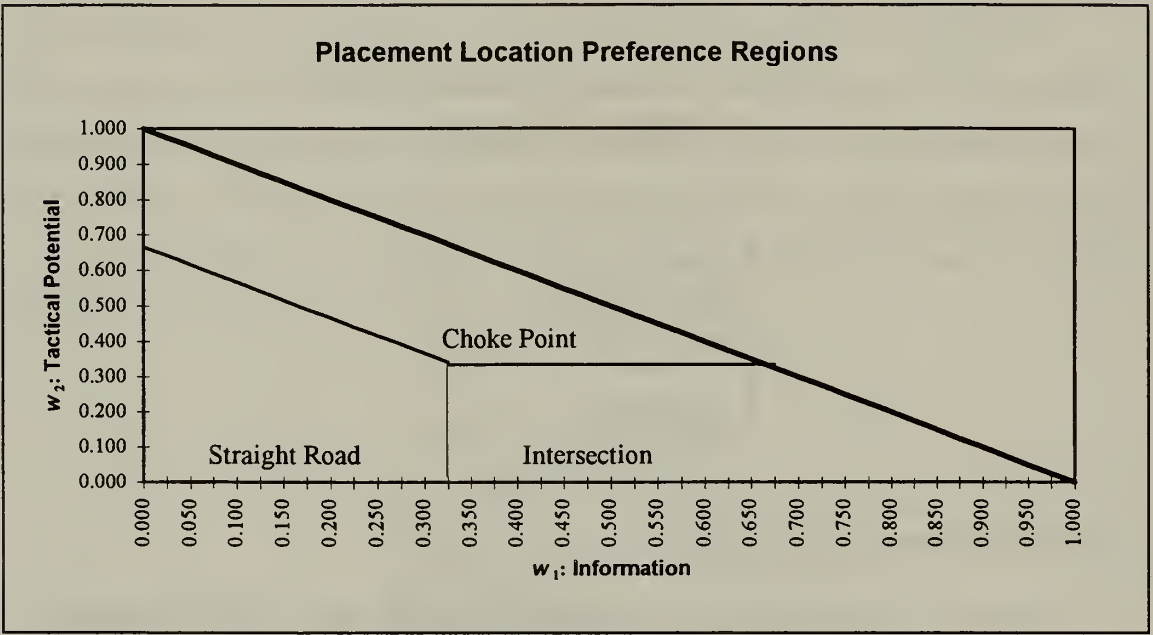

Figure 4.1 - Placement Location Preference Regions

For example, the intelligence analyst, who is most concerned with information flow, may consider the information attribute paramount while having little concern for the tactical potential of the array. In his case, the array is best located at an intersection. On the other hand, the strike pilot is only interested in his ability to localize a target identified by the array. His choice would be for the choke point. The final extreme is represented by the SOF planner concerned with providing useful information without compromising the insertion team. His policy of choice would be the straight road. The diagram also provides a representation of the case illustrated in Table 4.4. This decision maker, who weighs all attributes equally, draws no information from the policy diagram.

V. CONCLUSIONS

Used effectively, unattended ground sensors are a significant asset in the theater commander's TBMD toolbox. The key to accomplishing the optimal use of this resource, and those that rely upon it, is a step by step plan to follow for their employment. This thesis provides the checklist to be used by theater commanders to maximize ballistic missile defense at minimal cost.

A. THE PROCESS

The process begins with a specific theater with an overlying road grid. The theater commander is allotted N unattended ground sensors to be deployed at his discretion. The intelligence shop, after careful analysis selects A candidate array sites based on time critical target traffic and mission criticality. Provided with the list of sites is a corresponding P_x, the fraction of vehicular traffic along that segment believed to be time critical, and λ_i, the estimated vehicle flow rate.

The theater commander then enlists a member of his staff to run the optimization given in Chapter III, Section B to minimize his expected losses based on the above parameters. The nonlinear program produces an optimal array size, n^*, and prosecution policy, k^*, for each of the A sites. If nonlinear programming software is unavailable, Appendix B may be used in which the maximum allowable probability of a leaker is compared to that of a false alarm to obtain an n^*. Appendix C then provides the optimal policy on prosecution using the array sizes specified in Appendix B. These look up tables are generated by enumerating the possible combinations of n and k, then choosing that which produces the minimal loss by Equation (3.2). It is important to note that the look up table procedure may not provide the actual optimal solution for the case where N is limited. The look up tables merely supply the optimal array size for discrete roads, without limiting the total count of sensors deployed.

With the array sizes and policies determined, the next step is to find the inter-sensor spacing to be used given that it is desirous to minimize the reporting time between adjacent sensors and also minimize the likelihood that the entire array is compromised by hostile forces. Appendix D provides look up tables for sensor distance based on array size, estimated TCT speed, and the maximum reporting time for the array. When entering the look up tables, it is important to subtract one for each pair of sensors that report virtually simultaneously because of their pattern orientation, as described in Chapter III Section D.

The geometry of the pattern is devised through the theater commander's relative weight of reporting time, compromise, and susceptibility to counter-measures. These weights are applied to Figure 3.7 and the optimal deployment pattern is read from the graph. To maximize the array effectiveness according to this theater commander's desires, sensors should be placed in the appropriate pattern with the spacing determined above.

After the specifics of the *A* arrays are complete, it is time to begin mission planning. The theater commander must meet with his intelligence analysts and operational planners to determine the relative weights of the factors affecting array locations, placement and compromise, information, and tactical potential. These weights are then applied to Figure 4.1 to ascertain the type of location most beneficial to the overall effort, yet in an area conducive to array deployment. These locations are broken into the categories of choke points, straight roads, and intersections.

B. SUMMARY

As budgets continue to shrink and small theater actions become more common, optimal use of available assets exponentially increases in importance. The Vietnam era tradition of attrition warfare has given way to today's cost effective battlefield upon which fewer soldiers, and a large number of less expensive sensors are placed. This is the essence of Libicki's technological "Mesh" in which many small sensors perform all the data collection with the added advantage of being too numerous to kill, and thereby more robust. In fact, the role of unattended ground sensors can be summed up in that

> being there is necessarily a prerequisite to seeing there, and not necessarily a prerequisite to hitting there if the range set of one's own weapons is sufficiently dense.

Judiciously placed sensors, combined with lethal UAV's, artillery, or theater missiles would go a long way toward this vision. This document begins to satisfy the first portion of that equation.

Theater ballistic missiles pose an ominous threat to any theater commander in the battlefields of the future. It is only through the judicious use of all available assets that decisive action may be taken. Properly employed unattended ground sensors provide a cost effective and reliable option to assess the theater throughout the conflict.

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APPENDIX A. THE "STEEL RATTLER" SENSOR

As an excellent example of the advances in sensor technology, "Steel Rattler" unattended ground sensors are used as the basis for the analyses in this thesis. The capabilities of both the sensor units and their deployment systems continue to evolve, but this appendix serves as a current-day ability profile. The sensors were designed and tested by Sandia National Laboratories in Albuquerque, New Mexico, from which most of this information was obtained. Further data was provided by Central MASINT Technology of Florida.

The "Steel Rattler" is a multi-component unattended ground sensor system with seismic, acoustic and infrared detection and identification capability. The seismic/ acoustic array first detects a target of interest and attempts to match its signature with a pre-loaded signature database. The time of detection, array position, and identification are sent via satellite link to a fusion center. If a positive identification is not possible, the seismic/ acoustic array "wakes up" the infrared sensor which is positioned further along the expected route of travel. The infrared sensor transmits a still photograph of the target of interest at its closest point of approach to the fusion center via a satellite link where a system operator must visually identify the contact. If the seismic/ acoustic array makes an identification, the infrared sensor will never be activated. There is no ability to turn on the infrared to confirm the sensor's identification. Similarly, if the seismic/ acoustic array fails to detect a target of interest, the infrared sensor has no means to detect on its own. It is possible to position a seismic/ acoustic array on either side of the infrared sensor to detect targets moving in either direction.

The seismic/ acoustic array field of regard is 360°. Therefore, the search area for the seismic/ acoustic array is circular with a radius equal to the maximum seismic/ acoustic range centered at the array position. This maximum range is approximately 500 meters, depending on the specific terrain in which the array is placed. For the purpose of this analysis, all sensors are assumed to be "cookie-cutter," implying that there is no chance of detecting a target outside the specified maximum range. In reality, some detections may occur in this region.

The “Steel Rattler” can be described as a system performing spot searches on a recurring basis. The actual sensor sample rate is once per second, but a maximum of five seconds are required to report a detection to the fusion center. For this reason, a five second sample rate is used in the model analysis of Chapter III. The time required to check a target signature against the database is less than one second, and therefore considered negligible for this thesis.[Ref. 7]

APPENDIX B. SIZE LOOK-UP TABLES

The tables in this appendix are based on the k^* or more of n^* policy given in Chapter II and the derivation in Appendix F. They are subdivided by the relative values of a leaker versus a false alarm, given as r_1 and r_2 respectively. Each table has a corresponding P_x value which is the fraction of traffic assumed to be TCTs. The sensors are assumed identical with $f_1 = 0.80$ and $f_0 = 0.15$, as previously described. The table is entered with a row value of the maximum allowable probability of prosecuting a non-TCT, and a column value of the maximum allowable probability of a leaker.

Table entries are obtained by enumerating possible values of n for the appropriate probabilities of a leaker and of a false alarm, and choosing the minimum array size. To summarize this procedure, let $k(n)$ be the optimal policy which minimizes $l(n,k)$. Further, let

$\alpha(n, k)$ be the probability of a leaker, P(Leaker), and

$\beta(n, k)$ be the probability of prosecuting a non-TCT, P(Hit F.T.).

The optimal array size may then be obtained from the math program given by

$$
\begin{aligned}
&\textit{Minimize} \quad n \\
&s.t. \\
&\alpha(n,k(n)) \le \text{Max. allowable } P(\text{Leaker}) \\
&\beta(n,k(n)) \le \text{Max. allowable } P(\text{Hit F.T.}) \\
&n \ge 1 \\
&1 \le k \le n \\
&n,k \quad \textit{Integer}
\end{aligned}
$$

For Example, assume leakers and false alarms have relative values of $r_1 = 1$ and $r_2 = 1$, respectively. Now assume that the theater commander wants to know the optimal array size given that he will allow a 5% leaker probability and a 20% chance of prosecuting a non-TCT. The intelligence shop estimates that 20% of vehicle traffic are TCT's. Then, from the $P_x = 0.4$ Table in Section B, the optimal array size is $n^* = 3$.

A. "Leaker," $r_1 = 1$ and "False alarm," $r_2 = 1$

$P_x = 0.1$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	6	3	1	1	1	1
Max	0.05	6	2	1	1	1	1
P(Hit F.T.)	0.10	6	2	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.2$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	7	4	4	4	1	1
Max	0.05	7	3	2	2	1	1
P(Hit F.T.)	0.10	7	3	2	2	1	1
	0.15	7	3	2	2	1	1
	0.20	7	3	2	2	1	1
	0.25	7	3	2	2	1	1

$P_x = 0.3$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	6	4	4	4	4
Max	0.05	8	3	3	2	2	2
P(Hit F.T.)	0.10	8	3	3	2	2	2
	0.15	8	3	1	1	1	1
	0.20	8	3	1	1	1	1
	0.25	8	3	1	1	1	1

$P_x = 0.4$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	7	7	7	7	7
Max	0.05	6	3	2	2	2	2
P(Hit F.T.)	0.10	6	3	1	1	1	1
	0.15	6	3	1	1	1	1
	0.20	6	3	1	1	1	1
	0.25	6	3	1	1	1	1

$P_x = 0.5$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	7	7	7	7	7
Max	0.05	6	5	3	3	3	3
P(Hit F.T.)	0.10	6	4	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.6$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	7	7	7	7	7
Max	0.05	8	4	3	3	3	3
P(Hit F.T.)	0.10	8	4	3	1	1	1
	0.15	8	2	2	1	1	1
	0.20	8	2	2	1	1	1
	0.25	8	2	2	1	1	1

$P_x = 0.7$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	5	5	5	5	5
Max	0.05	7	4	3	1	1	1
P(Hit F.T.)	0.10	7	2	2	1	1	1
	0.15	7	2	2	1	1	1
	0.20	7	2	2	1	1	1
	0.25	7	2	2	1	1	1

$P_x = 0.8$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	6	6	6	6	6
Max	0.05	5	4	4	4	1	1
P(Hit F.T.)	0.10	3	2	2	2	1	1
	0.15	3	2	2	2	1	1
	0.20	3	2	2	2	1	1
	0.25	3	2	2	2	1	1

$P_x = 0.9$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	7	6	6	6	6	6
Max	0.05	3	2	2	2	2	2
P(Hit F.T.)	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

B. "Leaker," $r_1 = 2$ and "False alarm," $r_2 = 1$

$P_x = 0.1$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	6	6	6	6	6	6
Max	0.05	5	2	2	2	2	2
P(Hit F.T.)	0.10	5	2	2	2	2	2
	0.15	5	1	1	1	1	1
	0.20	5	1	1	1	1	1
	0.25	5	1	1	1	1	1

$P_x = 0.2$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	7	4	4	4	4	4
Max	0.05	7	3	2	2	2	2
P(Hit F.T.)	0.10	7	3	2	2	2	2
	0.15	7	1	1	1	1	1
	0.20	7	1	1	1	1	1
	0.25	7	1	1	1	1	1

$P_x = 0.3$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	7	7	7	7	7
Max	0.05	6	3	3	3	3	3
P(Hit F.T.)	0.10	4	3	3	3	3	3
	0.15	4	3	1	1	1	1
	0.20	4	2	1	1	1	1
	0.25	4	2	1	1	1	1

$P_x = 0.4$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	7	7	7	7	7
Max	0.05	6	3	3	3	3	3
P(Hit F.T.)	0.10	6	3	1	1	1	1
	0.15	6	3	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.5$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	10	7	7	7	7	7
Max	0.05	6	5	3	3	3	3
P(Hit F.T.)	0.10	6	4	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.6$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	8	8	8	8	8
Max	0.05	7	6	3	3	3	3
P(Hit F.T.)	0.10	5	4	3	1	1	1
	0.15	5	2	2	1	1	1
	0.20	5	2	2	1	1	1
	0.25	5	2	2	1	1	1

$P_x = 0.7$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	8	8	8	8	8
Max	0.05	5	4	4	4	4	4
P(Hit F.T.)	0.10	5	2	2	2	2	2
	0.15	3	2	2	2	2	2
	0.20	3	2	2	2	2	2
	0.25	3	2	2	2	2	2

$P_x = 0.8$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	6	6	6	6	6
Max	0.05	5	4	4	4	4	4
P(Hit F.T.)	0.10	3	2	2	2	2	2
	0.15	3	2	2	2	2	2
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

$P_x = 0.9$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	7	7	7	7	7	7
Max	0.05	3	2	2	2	2	2
P(Hit F.T.)	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

C. "Leaker," $r_1 = 1$ and "False alarm," $r_2 = 2$

$P_x = 0.1$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	6	3	1	1	1	1
Max	0.05	6	2	1	1	1	1
P(Hit F.T.)	0.10	6	2	1	1	1	1
	0.15	6	2	1	1	1	1
	0.20	6	2	1	1	1	1
	0.25	6	2	1	1	1	1

$P_x = 0.2$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	7	4	3	3	1	1
Max	0.05	7	4	2	2	1	1
P(Hit F.T.)	0.10	7	4	2	2	1	1
	0.15	7	4	2	2	1	1
	0.20	7	4	2	2	1	1
	0.25	7	4	2	2	1	1

$P_x = 0.3$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	6	4	4	4	4
Max	0.05	9	3	3	2	2	2
P(Hit F.T.)	0.10	9	3	3	2	2	2
	0.15	9	3	1	1	1	1
	0.20	9	3	1	1	1	1
	0.25	9	3	1	1	1	1

$P_x = 0.4$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	6	4	4	4	4
Max	0.05	9	3	3	2	2	2
P(Hit F.T.)	0.10	9	3	1	1	1	1
	0.15	9	3	1	1	1	1
	0.20	9	3	1	1	1	1
	0.25	9	3	1	1	1	1

$P_x = 0.5$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	9	6	4	4	4	4
Max	0.05	8	5	3	3	2	2
P(Hit F.T.)	0.10	8	5	1	1	1	1
	0.15	8	5	1	1	1	1
	0.20	8	5	1	1	1	1
	0.25	8	5	1	1	1	1

$P_x = 0.6$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	7	7	7	7	2
Max	0.05	8	4	3	3	3	2
P(Hit F.T.)	0.10	8	4	3	1	1	1
	0.15	8	4	3	1	1	1
	0.20	8	4	3	1	1	1
	0.25	8	4	3	1	1	1

$P_x = 0.7$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	5	5	5	5	5
Max	0.05	8	4	3	1	1	1
P(Hit F.T.)	0.10	8	2	2	1	1	1
	0.15	8	2	2	1	1	1
	0.20	8	2	2	1	1	1
	0.25	8	2	2	1	1	1

$P_x = 0.8$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	8	5	5	5	5	5
Max	0.05	8	4	3	3	1	1
P(Hit F.T.)	0.10	8	2	2	2	1	1
	0.15	8	2	2	2	1	1
	0.20	8	2	2	2	1	1
	0.25	8	2	2	2	1	1

$P_x = 0.9$

		Max P(Leaker)					
		0.01	0.05	0.10	0.15	0.20	0.25
	0.01	6	6	6	6	6	6
Max	0.05	3	2	2	2	2	2
P(Hit F.T.)	0.10	1	1	1	1	1	1
	0.15	1	1	1	1	1	1
	0.20	1	1	1	1	1	1
	0.25	1	1	1	1	1	1

APPENDIX C. POLICY LOOK-UP TABLES

The tables in this appendix give the optimal k^* of the "k or more of n" policy described in Chapter II. They are subdivided by the relative values of a leaker versus a false alarm, given as r_1 and r_2 respectively. Each table has a corresponding P_x value which is the local fraction of traffic assumed to be TCT. The sensors are assumed identical with $f_1 = 0.80$ and $f_0 = 0.15$. Table values are obtained by fixing P_x and n, then enumerating values of the loss function (Equation (3.2)) for varying k. The k corresponding to the minimum loss function value is k^*.

For Example, assume leakers and false alarms have relative values of $r_1 = 1$ and $r_2 = 2$, respectively. Now assume that the theater commander wants to know the optimal k or more of n^* policy given an array size of $n = 8$ sensors. The intelligence shop estimates that 40% of vehicle traffic are TCT's. Then, from the $P_x = 0.4$ column in Section B, the optimal value of k is $k^* = 4$. That is, the contact at the sensor should be prosecuted as a TCT if four or more of the eight sensors indicate that it is a TCT.

A. Leaker," $r_1 = 1$ and "False alarm," $r_2 = 1$

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	2	3	2
4	3	4	3	4	3
5	4	5	3	5	3
6	4	6	4	6	4
7	4	7	4	7	4
8	5	8	5	8	4
9	5	9	5	9	5
10	6	10	6	10	5

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	2	2	1	2	1
3	2	3	2	3	2
4	2	4	2	4	2
5	3	5	3	5	3
6	3	6	3	6	3
7	4	7	4	7	4
8	4	8	4	8	4
9	5	9	5	9	5
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	1	3	1
4	2	4	2	4	2
5	3	5	2	5	2
6	3	6	3	6	3
7	3	7	3	7	3
8	4	8	4	8	4
9	4	9	4	9	4
10	5	10	5	10	4

B. "Leaker," $r_1 = 2$ and "False alarm," $r_2 = 1$

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	2	2	2	2	1
3	2	3	2	3	2
4	3	4	3	4	2
5	3	5	3	5	3
6	4	6	4	6	3
7	4	7	4	7	4
8	5	8	4	8	4
9	5	9	5	9	5
10	6	10	5	10	5

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	2	3	2
4	2	4	2	4	2
5	3	5	3	5	2
6	3	6	3	6	3
7	4	7	4	7	3
8	4	8	4	8	4
9	5	9	4	9	4
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	1	2	1	2	1
3	1	3	1	3	1
4	2	4	2	4	1
5	2	5	2	5	2
6	3	6	3	6	2
7	3	7	3	7	2
8	4	8	4	8	3
9	4	9	4	9	3
10	5	10	4	10	4

C. "Leaker," $r_1 = 1$ and "False alarm," $r_2 = 2$

$P_x = 0.10$		$P_x = 0.20$		$P_x = 0.30$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	2
4	3	4	3	4	3
5	4	5	3	5	3
6	4	6	4	6	4
7	5	7	4	7	4
8	5	8	5	8	5
9	6	9	5	9	5
10	6	10	6	10	6

$P_x = 0.40$		$P_x = 0.50$		$P_x = 0.60$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	2	2	2	2	2
3	2	3	2	3	2
4	3	4	3	4	2
5	3	5	3	5	3
6	4	6	4	6	3
7	4	7	4	7	4
8	5	8	4	8	4
9	5	9	5	9	5
10	5	10	5	10	5

$P_x = 0.70$		$P_x = 0.80$		$P_x = 0.90$	
n	*k**	*n*	*k**	*n*	*k**
1	1	1	1	1	1
2	1	2	1	2	1
3	2	3	2	3	1
4	2	4	2	4	2
5	3	5	3	5	2
6	3	6	3	6	3
7	4	7	4	7	3
8	4	8	4	8	4
9	5	9	4	9	4
10	5	10	5	10	5

APPENDIX D. SPACING LOOK-UP TABLES

The tables in this appendix give the inter-sensor distance for array sensors as described in Chapter III. Most of the factors affecting sensor spacing are beyond the control of the theater commander, so these values are based on some simple assumptions. It is important to note that the number of sensors in the array for the look-up represent only the sensors not making simultaneous reports. In the case where w sensors report to the theater commander virtually simultaneously, only one of the w is used in computing the array size. Table values are obtained from Equation (3.4) with the assumption that TCT velocity is constant through the array. A separate table is provided for speeds varying from 5 to 55 kph, and for one to ten sensor array sizes.

For Example, assume that the theater commander wants to know the optimal inter-sensor distance given his array size of $n = 8$ sensors. The intelligence shop estimates that TCT's along this stretch of road travel at approximately $v = 30$ kph and the theater commander wants his full array to report in no more than $t_R = 6$ minutes. Then, from the "Speed $v = 30$ kph" table, reading the $n = 8$ column and the $t_R = 6$ minutes yields a maximum sensor spacing of $d = 375$ meters.

Distance for TCT Speed $v = 5$ kph

t_R	Array Size, n								
(min)	2	3	4	5	6	7	8	9	10
1	83	42	28	21	17	14	12	10	9
2	167	83	56	42	33	28	24	21	19
3	250	125	83	63	50	42	36	31	28
4	333	167	111	83	67	56	48	42	37
5	417	208	139	104	83	69	60	52	46
6	500	250	167	125	100	83	71	63	56
7	583	292	194	146	117	97	83	73	65
8	667	333	222	167	133	111	95	83	74
9	750	375	250	188	150	125	107	94	83
10	833	417	278	208	167	139	119	104	93

Distance for TCT Speed $v = 10$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	167	83	56	42	33	28	24	21	19
2	333	167	111	83	67	56	48	42	37
3	500	250	167	125	100	83	71	63	56
4	667	333	222	167	133	111	95	83	74
5	833	417	278	208	167	139	119	104	93
6	1000	500	333	250	200	167	143	125	111
7	1167	583	389	292	233	194	167	146	130
8	1333	667	444	333	267	222	190	167	148
9	1500	750	500	375	300	250	214	188	167
10	1667	833	556	417	333	278	238	208	185

Distance for TCT Speed $v = 15$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	250	125	83	63	50	42	36	31	28
2	500	250	167	125	100	83	71	63	56
3	750	375	250	188	150	125	107	94	83
4	1000	500	333	250	200	167	143	125	111
5	1250	625	417	313	250	208	179	156	139
6	1500	750	500	375	300	250	214	188	167
7	1750	875	583	438	350	292	250	219	194
8	2000	1000	667	500	400	333	286	250	222
9	2250	1125	750	563	450	375	321	281	250
10	2500	1250	833	625	500	417	357	313	278

Distance for TCT Speed $v = 20$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	333	167	111	83	67	56	48	42	37
2	667	333	222	167	133	111	95	83	74
3	1000	500	333	250	200	167	143	125	111
4	1333	667	444	333	267	222	190	167	148
5	1667	833	556	417	333	278	238	208	185
6	2000	1000	667	500	400	333	286	250	222
7	2333	1167	778	583	467	389	333	292	259
8	2667	1333	889	667	533	444	381	333	296
9	3000	1500	1000	750	600	500	429	375	333
10	3333	1667	1111	833	667	556	476	417	370

Distance for TCT Speed $v = 25$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	417	208	139	104	83	69	60	52	46
2	833	417	278	208	167	139	119	104	93
3	1250	625	417	313	250	208	179	156	139
4	1667	833	556	417	333	278	238	208	185
5	2083	1042	694	521	417	347	298	260	231
6	2500	1250	833	625	500	417	357	313	278
7	2917	1458	972	729	583	486	417	365	324
8	3333	1667	1111	833	667	556	476	417	370
9	3750	1875	1250	938	750	625	536	469	417
10	4167	2083	1389	1042	833	694	595	521	463

Distance for TCT Speed $v = 30$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	500	250	167	125	100	83	71	63	56
2	1000	500	333	250	200	167	143	125	111
3	1500	750	500	375	300	250	214	188	167
4	2000	1000	667	500	400	333	286	250	222
5	2500	1250	833	625	500	417	357	313	278
6	3000	1500	1000	750	600	500	429	375	333
7	3500	1750	1167	875	700	583	500	438	389
8	4000	2000	1333	1000	800	667	571	500	444
9	4500	2250	1500	1125	900	750	643	563	500
10	5000	2500	1667	1250	1000	833	714	625	556

Distance for TCT Speed $v = 35$ kph

t_R (min)	Array Size, *n*								
	2	3	4	5	6	7	8	9	10
1	583	292	194	146	117	97	83	73	65
2	1167	583	389	292	233	194	167	146	130
3	1750	875	583	438	350	292	250	219	194
4	2333	1167	778	583	467	389	333	292	259
5	2917	1458	972	729	583	486	417	365	324
6	3500	1750	1167	875	700	583	500	438	389
7	4083	2042	1361	1021	817	681	583	510	454
8	4667	2333	1556	1167	933	778	667	583	519
9	5250	2625	1750	1313	1050	875	750	656	583
10	5833	2917	1944	1458	1167	972	833	729	648

Distance for TCT Speed $v = 45$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	750	375	250	188	150	125	107	94	83
2	1500	750	500	375	300	250	214	188	167
3	2250	1125	750	563	450	375	321	281	250
4	3000	1500	1000	750	600	500	429	375	333
5	3750	1875	1250	938	750	625	536	469	417
6	4500	2250	1500	1125	900	750	643	563	500
7	5250	2625	1750	1313	1050	875	750	656	583
8	6000	3000	2000	1500	1200	1000	857	750	667
9	6750	3375	2250	1688	1350	1125	964	844	750
10	7500	3750	2500	1875	1500	1250	1071	938	833

Distance for TCT Speed $v = 55$ kph

t_R (min)	Array Size, n								
	2	3	4	5	6	7	8	9	10
1	917	458	306	229	183	153	131	115	102
2	1833	917	611	458	367	306	262	229	204
3	2750	1375	917	688	550	458	393	344	306
4	3667	1833	1222	917	733	611	524	458	407
5	4583	2292	1528	1146	917	764	655	573	509
6	5500	2750	1833	1375	1100	917	786	688	611
7	6417	3208	2139	1604	1283	1069	917	802	713
8	7333	3667	2444	1833	1467	1222	1048	917	815
9	8250	4125	2750	2063	1650	1375	1179	1031	917
10	9167	4583	3056	2292	1833	1528	1310	1146	1019

APPENDIX E. PROBABILITY OF IDENTIFICATION

Chapter II introduces the probability of identification calculations used within the models. These probabilities are based on conditional probability and derived from both Bayes' Rule and the Law of Total Probability. This appendix shows the derivation of the exactly k of n (Equation [2.3]) and the k or more of n (Equation [2.4]) computations.

A. SINGLE SENSOR ARRAY

The simplest case is an array consisting of a single sensor placed along a road. For the purposes of this analysis, it will still be considered an "array," and in fact will be the basic building block of all larger arrays. This array, as with all the arrays to be discussed, provides decision probabilities, $P(\text{TCT} \mid i/1)$, based on the sensor output and the fraction of vehicle traffic assumed to be TCT. $P(\text{TCT} \mid 1/1)$ is defined as the probability that the target is a TCT given the sensor reports it as TCT. Similarly, $P(\text{TCT} \mid 0/1)$ is the probability that the target is TCT given the sensor reports it as non-TCT . In all cases, P_x, the fraction of vehicles assumed to be TCT, must be provided by some intelligence estimate.[Ref. 4]

Let

$$X = \begin{cases} 1 \text{ if the target moving past the sensor is a TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

Each sensor outputs a forecast denoted by:

$$F_i = \begin{cases} 1 \text{ if the sensor identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

Additional sensors are denoted using increasing subscripts, i.e. $i=1, 2, 3, \ldots$

From sensor performance data provided by the manufacturer and field tests, the forecast likelihoods are given by

f_1 = P{Sensor indicates a TCT given a TCT present}, and

f_0 = P{Sensor indicates a TCT given no TCT present}.

That is,

$$f_1 = P\{F = 1 | X = 1\}, \text{ and}$$

$$f_0 = P\{F = 1 | X = 0\}.$$

For the simple one sensor case, using the forecast probabilities and Bayes' Rule, the decision probabilities are [Ref. 4]:

$$P(\text{TCT}|1/1) = \frac{f_1 \cdot P_x}{f_1 \cdot P_x + f_0(1 - P_x)},$$

$$P(\text{TCT}|0/1) = \frac{(1 - f_1) \cdot P_x}{(1 - f_1) \cdot P_x + (1 - f_0)(1 - P_x)}.$$

B. TWO SENSOR ARRAY

The two sensor array consists of two sensors spaced close enough to assume both identification calls reported to the fusion center are on the same target. The theater commander will be provided with the decision probabilities, P(TCT | k/n). As before, the estimate, P_x, must be provided by intelligence.

Each of the two sensors will output a forecast denoted by

$$F_1 = \begin{cases} 1 \text{ if sensor 1 identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

$$F_2 = \begin{cases} 1 \text{ if sensor 2 identifies the target as TCT,} \\ 0 \text{ otherwise.} \end{cases}$$

If the sensors are assumed to be conditionally independent, then

$$P\{F_1 = i_1, F_2 = i_2 | X = x\} = P\{F_1 = i_1 | X = x\} P\{F_2 = i_2 | X = x\}, \quad \text{where } i, x \in \{0, 1\}.$$

Also, since the sensors are identical, $P(F_1 = f \mid X = x) = P(F_2 = f \mid X = x)$ for every f and x. Therefore $f_{1(1)} = f_{1(2)}$ and $f_{0(1)} = f_{0(2)}$. Henceforth, f_1 and f_0 will be used for the forecast likelihoods of all identical sensors.

The decision probabilities are given by [Ref. 4]

$$P(\mathrm{TCT}|2/2) = \frac{f_1^2 P_x}{f_1^2 P_x + f_0^2 (1 - P_x)},$$

$$P(\mathrm{TCT}|1/2) = \frac{(1 - f_1) f_1 P_x}{(1 - f_1) f_1 P_x + (1 - f_0) f_0 (1 - P_x)}, \text{ and}$$

$$P(\mathrm{TCT}|0/2) = \frac{(1 - f_1)^2 P_x}{(1 - f_1)^2 P_x + (1 - f_0)^2 (1 - P_x)}.$$

C. GENERAL FORMALATION, n ≥ 2

Similar results hold for the general case involving $n \geq 2$ sensors. Again, assuming conditional independence allows

$$P\{F_1 = i_1, \ldots, F_n = i_n | X = x\} = P\{F_1 = i_1 | X = x\} \cdot \ldots \cdot P\{F_n = i_n | X = x\},$$

where i_n, $x \in \{0,1\}$. As above, the arrays are composed of identical sensors, and therefore

$$P\{F_1 = 1 | X = 1\} = P\{F_n = 1 | X = 1\} = f_1, \text{ and}$$

$$P\{F_1 = 1 | X = 0\} = P\{F_n = 1 | X = 0\} = f_0.$$

Now, let k equal the number of sensors in an array of size n to identify a passing target as a TCT. From Bayes' Rule and the Law of Total Probability the decision probabilities for exactly k of n sensors indicating a TCT are given by

$$P(\mathrm{TCT}|k/n) = \frac{f_1^k (1 - f_1)^{n-k} P_x}{f_1^k (1 - f_1)^{n-k} P_x + f_0^k (1 - f_0)^{n-k} (1 - P_x)}.$$

This equation can be used for any array size with a given intelligence estimate of the TCT population.

D. FINDING *k* OR MORE OF *n* POLICY

Again assume that an array of n identical sensors is in place with forecast likelihoods of f_1 and f_0. From Section C the exact k of n decision probability is given as

$$P(\mathrm{TCT}|k/n) = \frac{f_1^k(1-f_1)^{n-k} P_x}{f_1^k(1-f_1)^{n-k} P_x + f_0^k(1-f_0)^{n-k}(1-P_x)}.$$

APPENDIX F. *k* OR MORE OF *n* AND LOSS FUNCTION COMPUTATIONS

The "at least *k*-out-of-*n*" policy of Chapter II and the loss function introduced in Chapter III are the driving forces behind the two models used in this thesis. Their roots are based in decision theory and conditional probability. This appendix shows a complete derivation of the *k* or more of *n* policy, and how it is used to generate the loss function, $l(n,k)$, of the optimization.

The decision required in this thesis is to choose an integer k, where $0 \le k \le n$, such that the theater commander will take action if and only if at least k sensors indicate a vehicle is a TCT. Recall that r_1 is the loss obtained if no action is taken and the vehicle is a TCT, and r_2 is the loss obtained if a non-TCT is acted against. The expected loss is

$$\begin{aligned}
l(n,k) &= r_1\left(P\{X=1,S=0\}+P\{X=1,S=1\}+\ldots+P\{X=1,S=k-1\}\right)+ \\
&\qquad r_2\left(P\{X=0,S=k\}+P\{X=0,S=k+1\}+\ldots+P\{X=0,S=n\}\right) \\
&= r_1 P\{X=1,S\le k-1\}+r_2 P\{X=0,S\ge k\} \\
&= r_1 P\{S\le k-1, X=1\}P_x + r_2 P\{S\ge k, X=0\}(1-P_x) \\
&= r_1 B(k-1,n,f_1)P_x + r_2\left(1-B(k-1,n,f_0)(1-P_x)\right)
\end{aligned}$$

where $B(k_i,n,f_0)$ is the binomial distribution function given by $\sum_{j=0}^{k_i}\binom{n}{j} f_0^j(1-f_0)^{n-j}$.

Figure F.1 shows a Microsoft Excel v7.0 spreadsheet programmed to perform the above calculations in an interactive manner.

Given Input Sensor Array Data	
f1 = 0.8	0.2 = 1 - f1
f0 = 0.15	0.85 = 1 - f0
n =	9
Px =	0.15
r1	2
r2	1

Calculated Output										
k	D:Act	D: Not Act	Policy	D:Act	D: Not Act	Policy	L*	P(r1)	P(r2)
1	0.3404	0.3000	'N'	0.1969	0.0000	'N'	0.3000	0.1500	0.0000
2	0.1197	0.3000	'A'	0.5096	0.0000	'N'	0.1197	0.0000	0.3404
3	0.0288	0.2999	'A'	0.7303	0.0001	'N'	0.0289	0.0000	0.1197
4	0.0048	0.2991	'A'	0.8212	0.0009	'N'	0.0057	0.0005	0.0288
5	0.0005	0.2941	'A'	0.8452	0.0059	'N'	0.0064	0.0029	0.0048
6	0.0000	0.2743	'A'	0.8495	0.0257	'N'	0.0257	0.0128	0.0005
7	0.0000	0.2215	'A'	0.8500	0.0785	'N'	0.0785	0.0393	0.0000
8	0.0000	0.1309	'A'	0.8500	0.1691	'N'	0.1691	0.0846	0.0000
9	0.0000	0.0403	'A'	0.8500	0.2597	'N'	0.2597	0.1299	0.0000

Figure F.1 - Microsoft Excel Program

The numerical values on the table, with the exception of the probability columns represent the relative losses with the values of r_1, r_2, P_x, n and k given. Columns two through four represent the condition that k or more sensors indicate a TCT, while columns five through seven represent the condition that fewer than k do so. Finally, columns nine and ten are the probabilities that a "leaker" or a "false alarm" occur with the values given. It is these values, combined with the losses in column eight that provide the data for the look up tables in the earlier appendices.

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